Radiation and Chemical Reaction Effects On MHD Boundary Layer Flow Over An Exponentially Stretching Surface

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ABSTRACT

This study was conducted to investigate radiation and chemical reaction effects on MHD boundary layer flow in porous media over an exponentially stretching surface. In the paper, boundary layer flow, heat and mass transfer of incompressible viscous fluid over a porous stretching vertical surface in the presence of thermal radiation. A similarity transformation was used to reduce the governing system of PDEs to a set of nonlinear ordinary differential equations which are solved numerically using the fourth order Runge-kutta method with shooting technique. The numerical computations were presented in tabular and graphical forms for various fluid parameters controlling the fluid flow, heat and mass transfer. It shows that an increase in radiation produced a rise on the velocity, temperature and concentration profiles. Skin friction, Nusselt and Sherwood numbers increased with increasing radiation.

Keywords: Thermal, chemical reactions, numerical computation, shooting technique.

Aims Research Journal Reference Format:
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1. INTRODUCTION

In boundary layer phenomena, radiation and chemical effects on MHD fluid flow, heat and mass transfers in porous media is significant because of its influence past exponentially stretching surface. The investigation such as this can be modeled and solved experimentally, analytically or numerically. In doing this, the existing models are either modified or adjusted to reflect the current trends in academics as well as industries that make use of such research reports. Sharman (2004) studied unsteady effect on MHD free convective and mass transfer flow through porous medium with constant suction and constant heating flux in rotating system, whereas in convective fluid, when the flow of mass is caused by a temperature difference, thermal diffusion effects cannot be neglected. Srihari, Amand, Kishan (2006) investigated unsteady free convection flow of a viscous incompressible fluid past an infinite vertical plate with constant heat flux. In their study, viscous dissipative heat was taken into account under the influence of transverse magnetic field.

Nalinakshi, Dinesh and Chandrashekar (2013) related the effect of variable fluid properties and MHD on mixed convection heat transfer from a vertically heated plate embedded in a sparsely packed porous medium. Sekhar (2014) carried out analysis on the boundary layer phenomena of MHD flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. Unsteady MHD flow of a visco-elastic fluid along vertical porous surface with fluctuating temperature and concentration was presented by Mohanty, Rath and Dash (2014). Also, Ibrahim and Suneetha (2015) presented the effects of heat generation and thermal radiation on steady MHD flow near a stagnation point on a stretching sheet in porous medium and presence of variable thermal conductivity and mass transfer, it was discovered that temperature increased with increasing radiation parameter R and concentration decreased with increasing Schmidt number.

Unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with thermal radiation and chemical reaction was recently analysed by Srivas and Kishan (2015). The effect of magnetic field was diminution with velocity of fluid flow in their study. Hussain and Ahmad (2015) verified the unsteady MHD flow and heat transfer for Newtonian fluids over an exponentially stretching sheet. Heat source and transfers are important factors in boundary because of universality researchers have extensively examined the combination of heat effects using various parameters on MHD among others. Heat flow and transfer of fluid through porous media is equally necessary as a result, the effects over stretching or shrinking surface are important.

As a result, Darcy parameter is very important when dealing with a study that involves porosity. Darcy's law is a constitutive equation that describes the flow of a fluid through a porous medium. The law is formulated by Henry Darcy based on the results of experiments on the flow of water through beds of sand, forming the basis of hydrogeology, a branch of earth sciences. Validity for laminar flow through sediments in fine ground sediments, the dimensions of insecticides are small and thus flow is laminar. Darcy porosity parameter is useful in petroleum engineering and geology. Like other permeability (measures) its dimensional units in length (Arora, 1989).

Prandtl number is another dimensionless number defined as the ratio of momentum diffusivity to thermal diffusivity. It is given as momentum diffusivity (kinetic viscosity). It is often used in heat transfer both in free and forced convection calculations. It involves various preparation of the fluid such as velocity, pressure. Prandtl gives information about the type of fluid, it talks about the thermal and hydrodynamics boundary layer. Nusselt number is the rate of heat transfer in terms of the Nusselt number at the plate (Nu). N_u gives comparison between conduction and convection heat transfer rate. Thermal radiation is a quantized electromagnetic radiation excited by thermal agitation of molecules or atoms and having a range including infrared visible light and ultraviolet. Sherwood number (mass transfer, Nusselt number in heat transfer) is a dimensionless number used in mass transfer operation.

It represents the ratio of the total rate of mass to the rate of diffusive mass transport alone. Schmidtl Number is a dimensionless parameter representing the ratio of diffusion of momentum to the diffusion of mass in a fluid. It is the mass transfer Prandtl number for gases Sc. and Pr. have similar values approximately (0.7) and this is used as basis for single heat and mass transfer analogies. It can be further defined as a function of Reynolds and Schmidtl numbers. Chemical reaction is a process that involves rearrangement of the molecules or ionic structure of a substance, as distinct in physical form or nuclear reaction. A process in which one or more substances the reactants are converted to one or more different substances, as in products. Substances are either chemical compounds or elements. A chemical reaction rearranges the constituent atoms of the reactants to create different substances as product. All these parameters have important bearings especially when studies involving heat and mass transfer in porous media are instituted.

In view of the above studies, the present study examine the radiation and chemical effects on heat and mass transfer of MHD in porous media over an exponentially stretching surface. Using suitable transformations, the required ODE of the third order corresponding to momentum equation, a second order DE corresponding to heat equation and a second order DE corresponding to mass transfer equation are solved; using Runge-Kutta method with shooting techniques. The numerical computations are performed to the desired level of accuracy. These were done for different values of dimensionless parameters of the model equations. The study considered these for the purpose of illustrating the research in tabular and graphical forms.

2. FORMULATION OF THE PROBLEM

Considering a free convective, the boundary layer flow, heat and mass transfer of incompressible fluid over an exponentially stretching surface. The emerging out of a slit at origin and moving with non-uniform velocity in the presence of thermal radiation and chemical reaction

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon}{K}u-\frac{\sigma}{\rho}B_0 u \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\gamma(C-C_\infty) \tag{4}$$

Subject to the following corresponding boundary conditions:

$$u=U_0 e^{\frac{x}{L}},\quad v=-V_0 e^{\frac{x}{L}}, T=T_w=T_\infty+T_0 e^{\frac{x}{2L}}, C=C_w=C_\infty+C_0 e^{\frac{x}{2L}} \text{ at y =0}$$
$$v\to 0,\quad T\to T_\infty,\ C\to C_\infty \quad as \quad y\to\infty \tag{5}$$

Where

u, v, *C* and *T* are velocity component in the *x* and *y* directions, concentration of the fluid species, fluid temperature respectively. *L* is the reference length, U_0 is the reference velocity, $V_0$is the permeability of the porous surface. The physical quantities K, *ρ*, *v*, *D*, *k*, C_p, and γ are the permeability of the porous medium, density, fluid kinematics viscosity, coefficient of mass diffusivity, thermal conductivity of the fluid, rate of specific internal heat generation or absorption and coefficient of mass diffusivity and reaction rate coefficient. q_r is the radiative heat flux in the *y* direction. By using the Rosseland approximation, Ibrahim and Suneetha (2015). The radiative heat flux q_r is given by:

$$q_r=-\frac{4\sigma_0\partial T^4}{3\delta\partial y} \tag{6}$$

Where

σ_0 and *δ* are the Stefan-Boltzmann approximation and the mean absorption coefficient respectively. Assume the temperature difference within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature, using Taylor series to expand T^4 about the free stream T_∞ and neglecting higher order terms, this gives the approximation

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{7}$$

The magnetic field $B(x)$ is assumed to be in the form $B(x)=B_0 e^{\frac{x}{2L}}$. Where B_0 is the constant magnetic field.

Introducing the stream function $\psi(x, y)$ such that $u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$ (8)

In this case when (8) is substituted in (1) continuity equation is identically satisfied and equations (2)-(4) become

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \upsilon \frac{\partial^3 \psi}{\partial y^3} - \frac{\sigma}{\rho} B_0 e^{\frac{x}{2L}}\left(\frac{\partial \psi}{\partial y}\right) - \frac{\upsilon}{k}\left(\frac{\partial \psi}{\partial y}\right) \tag{9}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \left(\frac{k}{\rho C_p} + \frac{16\sigma_0 T_\infty^3}{3\rho C_p \delta}\right)\frac{\partial^2 T}{\partial y^2} \tag{10}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \gamma(C - C_\infty) \tag{11}$$

The corresponding boundary conditions become:

$$\frac{\partial \psi}{\partial y} = U_0 e^{\frac{x}{L}}, \quad \frac{\partial \psi}{\partial x} = V_0 e^{\frac{x}{L}}, T = T_w = T_\infty + T_0 e^{\frac{x}{2L}}, C = C_w = C_\infty + C_0 e^{\frac{x}{2L}} \text{ at } y = 0$$

$$\frac{\partial \psi}{\partial y} \to 0, \quad T \to T_\infty, \; C \to C_\infty \quad as \quad y \to \infty \tag{12}$$

In order to transform the equations (9), (10) and (11) as well as the boundary conditions (12) into an ordinary differential equations, the following similarity transformations variables are introduced; (Sajid and Hayat, 2008).

$$\psi(x, y) = \sqrt{2\nu U_0 L} e^{\frac{x}{2L}} f(\eta), \eta = y\sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}}, T = T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta)$$

$$C = C_\infty - C_0 e^{\frac{x}{2L}} \phi(\eta) \tag{13}$$

Substituting the similarity variables (13) into equations (9), (10) and (11) the equations become

$$f''' + ff'' - 2f'^2 - (M + Da)f' = 0 \tag{14}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + P_r f\theta' - P_r f'\theta = 0 \tag{15}$$

$$\phi'' + Scf\phi' - Scf'\phi - Sc\lambda\phi = 0 \tag{16}$$

The corresponding boundary conditions take the form:

$$f' = 1, \theta = 1, \;\; \phi = 1, \; at \; \eta = 0, \; f' = 0, \theta' = 0, \;\; \phi' = 0 \;\; at \;\; \eta \to \infty \tag{17}$$

Where $M = \frac{2\sigma B_0^2}{\rho U_0}$ magnetic parameter, $Da = \frac{2\nu L}{U_0 K} e^{-\frac{x}{L}}$, $\Pr = \frac{\rho \upsilon C_p}{k}$ is the Prandt number, $R = \frac{4\sigma_0 T_\infty^3}{\delta k}$ is the thermal radiation parameter, $Sc = \frac{\upsilon}{D}$ is the Schmidt number $\lambda = \frac{2L\gamma}{U_0} e^{-\frac{x}{L}}$ is the chemical reaction parameter.

3. METHOD OF SOLUTION

The equations (14), (15) and (16) are non-linear coupled differential equations and its satisfying the boundary conditions (17). In order to solve the problems, the systems were reduced to nonlinear ordinary differential equations together with the boundary conditions were simplified and solved numerically using fourth order Runge-Kutta scheme with a shooting technique. The method was chosen because of its flexibility among the other methods. The following steps were applied: (i) The method replaces the given BVP by a sequence of Initial Value Problems (IVPs) for the same ODE with initial conditions. (ii) Integrating the sequence of (IVPs) using fourth order Runge-Kutta scheme. (iii) Identifying the initial slopes for the missing conditions using Newton-Raphson method. (iv) The integration length varies with the parameter values and it has been suitably chosen each time such that the boundary conditions at the outer edge of boundary layer are satisfied. (v) Repeating this procedure till the convergence is obtained satisfying the boundary conditions. Therefore, in order to integrate equations (14) - (16) as an initial value problem, the values for $f''(0)$, $\theta'(0)$ and $\phi'(0)$ were required but no such values were given in the boundary. The suitable guess values $f''(0)$, $\theta'(0)$ and $\phi'(0)$ were chosen and then integration was carried out. The researcher compared the calculated values for $f''(0)$, $\theta'(0)$ and $\phi(0)$ at $\eta = 8$ with the given boundary condition $f''(8) = 0$, $\theta'(8) = 0$ and $\phi(8) = 0$ Then adjusted the estimated values, $f''(0)$, $\theta'(0)$ and $\phi'(0)$, to give a better approximation for the solution. The researcher performed the series of values for $f''(0)$, $\theta'(0)$ and $\phi'(0)$, and then applied a fourth-order Runge–Kutta method with shooting techniques with step-size $h = 0.01$. The above procedure was repeated until the results up to the desired degree of accuracy 10^6. The following parameter values were adopted for computation as default number R=1,Sc.=0.62, *λ=0.5, Da= 0.1, Pr. =3.0, M=1.0.* All graphs correspond to the value except otherwise indicated on the graph.

Table 1: Effect of *M, R*, *Sc. Pr. Da* and λ on $f''(0), \theta'(0)$ and ϕ'(0) (P-Parameters)

P	Values	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	*P*	*Values*	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
M.	1.0	-1.7767	0.7742	0.8750	Pr.	3	-1.7767	0.7742	0.8750
	1.2	-1.8323	0.7470	0.8694		3.5	-1.7767	0.9038	0.8750
	1.3	-1.8594	0.7344	0.8668		4.0	-1.7767	1.0329	0.8750
	1.4	-1.8862	0.7224	0.8642		4.5	-1.7767	1.1605	0.8750
	1.5	-1.8995	0.7167	0.8629		5.0	-1.7767	1.2852	0.8750
R	1.0	-1.7667	0.7742	0.8750	λ	0.5	-1.7767	0.7742	0.8750
	3.0	-1.7667	0.3602	0.8750		3	-1.7767	0.7742	1.56 83
	5.0	-1.7667	0.2347	0.8750		7	-1.7767	0.7742	2.2387
	7.0	-1.7667	0.1740	0.8750		10	-1.7767	0.7742	2.6269
	10.0	-1.7667	0.1541	0.8750		12	-1.7767	0.7742	2.8556
Sc.	0.62	-1.6898	0.8213	0.8842	Da	0.1	-1.6597	0.8389	0.8875
	0.64	-1.6898	0.8213	0.9010		0.2	-1.6898	0.8213	0.8842
	0.66	-1.6898	0.8213	0.9175		0.3	-1.7192	0.8047	0.8810
	0.68	-1.6898	0.8213	0.9339		0.4	-1.7482	0.7890	0.8780
	0.70	-1.6898	0.8213	0.9500		0.5	-1.7625	0.7815	0.8765

4. RESULTS AND DISCUSSION OF THE FINDINGS

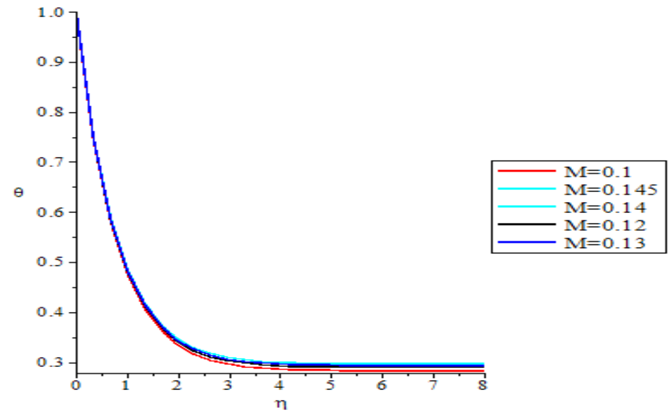
Table 1 presented the results based on the parameters of investigations. An increase in radiation parameter decreases the Nusselt number. It has no effects on skin friction and Sherwood number. The effect of varied Magnetic parameter was noticed. An increase in M leads to increase in Nusselt number and Sherwood number but decrease in skin friction. Chemical reaction has no effect on velocity and temperature profiles but has effect on concentration profiles. Pr. Number has no effect on velocity profile and concentration profile. An increase in Pr. Leads to corresponding increase in Nusselt number. This means that Pr. has effect on temperature profile. Effect of Darcy porosity was noticed in velocity, temperature and concentration profiles. An increase in Darcy porosity increases Nusselt and Sherwood numbers but decrease in Skin friction. Sc. has no effect on velocity and temperature profiles but its variation increases the level of concentration profile.

Figures 1-6 show the variations of the radiation and chemical reaction parameters on velocity, temperature and concentration profile. This is done to enable the researcher to determine the effectiveness of those parameters. It is observed that as λ increases, the concentration decreases as shown in figure 6. We also noticed that λ has no effect on velocity and temperature profile due to the appearance of the value of λ in equation (4). The values of Pr. increase as the temperature reduces which led to the decrease in boundary layer as shown in figure 6 but Pr. has no effect on the velocity and concentration profile as the values of Pr. is prominent in air than that of water due to the fact that water is more volatile than air.

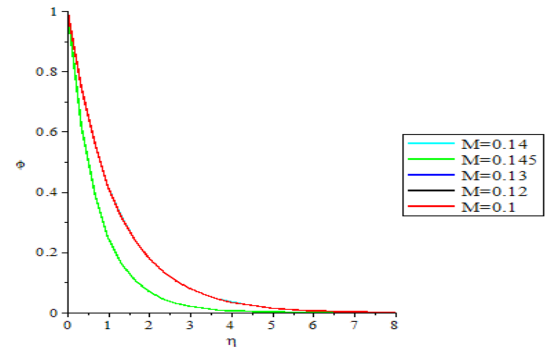
5. CONCLUSION

In conclusion, the study presented the behaviour of the dimensionless parameters in analysing the effect of radiation and chemical reaction of MHD boundary layer flow over an exponentially stretching porous surface. Using the similarity transformation to arrive at a set of ordinary differential equations was obtained from the governing equations. The momentum equation and the equation of concentration were solved numerically using maple 18. The results showed that the porous or permeability parameter reduced the rate of flow from the wall. An increase in magnetic field and Darcy parameters decrease exponential velocity, temperature and concentration profiles. Increase in Pr number increases temperature profile but decreases velocity and concentration profiles. An increase in radiation leads to decrease in temperature but no effect on velocity and concentration. An increase in Sc and λ lead to corresponding increase concentration but they do not have effects on velocity and temperature.

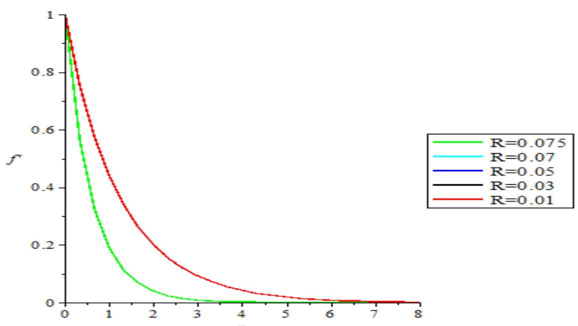
Figures



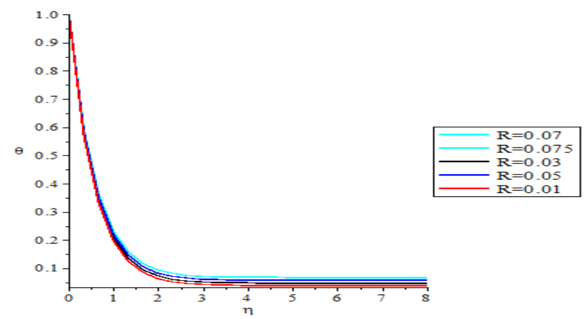
1. Temperature profile for varied *M*



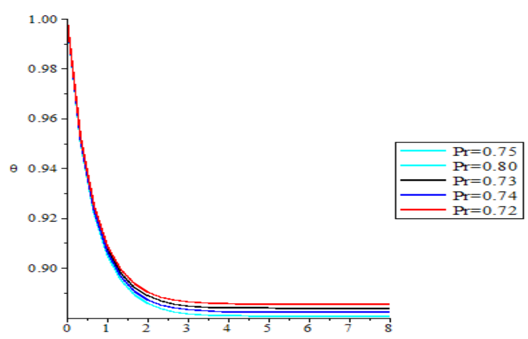
2. Concentration profile for varied *M*



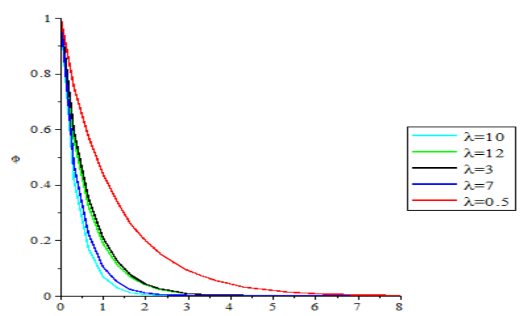
3. Velocity profile for varied *R*



4. Temperature profile for varied *R*



5. Temperature profile for varied *Pr*



6. Concentration profile for varied λ

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