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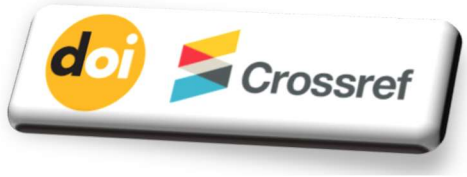

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# Utilization of Secondary School Students' Geometrical Learning Outcome and Experiences Through Hypothetical Learning Trajectory In Delta State

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## ABSTRACT

This study explores the application of Hypothetical Learning Trajectories (HLTs) in improving the teaching and learning of geometry among secondary school students in Delta State, Nigeria. Utilizing a thematic qualitative research design, the study examines how students' progress through stages of geometric learning, the challenges they encounter, and the strategies they employ to solve problems. Data were collected from 200 students across eight schools through classroom observations and analysis of work samples. Findings reveal four distinct stages in students' geometric learning: Initial Understanding, Exploration and Identification, Application and Analysis, and Mastery and Problem Solving. Each stage underscores the gradual deepening of conceptual and problem-solving abilities. Challenges such as abstract understanding, cognitive overload, and difficulties translating visuals into mathematical language were identified, emphasizing the need for adaptive teaching strategies. Students employed diverse problem-solving strategies, including visualization, step-by-step approaches, and collaborative learning. These methods align with global pedagogical best practices and highlight the potential of HLTs to foster deeper understanding and engagement. However, limitations such as reliance on trial-and-error and resource constraints underscore areas for improvement. The study concludes that HLTs enhance students' conceptual clarity, problem-solving confidence, and engagement in geometry. Recommendations include integrating visual aids, promoting collaborative exercises, employing hands-on and real-world applications, and leveraging digital tools to address cognitive challenges and resource limitations. By adopting these strategies, educators can create a more inclusive and effective geometry learning environment, contributing to improved student outcomes and broader pedagogical innovation.

**Keywords:** Utilization, Secondary School Students, Geometrical Learning Outcome, Experiences Hypothetical Learning Trajectory, Delta State, Nigeria

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## 1. INTRODUCTION

The teaching and learning of geometry in secondary schools play a crucial role in students' cognitive development and mathematical competency. Geaaometry not only serves as a foundation for excelling in mathematical exams but also equips students with essential problem-solving skills applicable to real-world contexts (Adeyemi & Adebayo, 2020). Despite its significance, student performance in geometry often shows notable disparities due to factors such as ineffective teaching methods, rigid curriculum designs, and varying levels of student engagement. Recent research suggests that aligning teaching approaches with students' developmental stages and prior knowledge can enhance learning outcomes (Natarajan, Smith, & Johnson, 2022). One promising pedagogical framework is the Hypothetical Learning Trajectory (HLT), which combines clearly defined learning goals, instructional strategies, and adaptive assessments. The HLT framework enables teachers to anticipate challenges, respond to individual learning needs, and foster a more tailored learning environment (Stein, Engel, & Smith, 2020). Globally, HLT has improved understanding and engagement in mathematical concepts, including geometry. However, its application in Nigeria, particularly in secondary schools, remains underexplored, creating an opportunity to investigate how this framework can address local challenges in geometrical education.

Geometry is a key component of the secondary school mathematics curriculum and is essential for further studies in mathematics, science, and engineering (Ajiboye, 2019). Proficiency in geometry demands skills such as spatial reasoning, understanding geometric principles, and problem-solving. However, in Nigeria, students often face low achievement rates in mathematics, including geometry. Contributing factors include traditional teaching methods that emphasize rote memorization over conceptual understanding and a lack of resources in many schools (Ogunyemi, 2021). These issues are compounded by the abstract nature of geometry, which requires complex visualization and spatial reasoning (Adeyemi & Adebayo, 2020). Traditional classroom settings frequently teach geometric concepts in isolation, limiting connections to students' prior knowledge and real-world applications. This fragmented approach can hinder students' grasp of fundamental principles and negatively impact their performance (Natarajan et al., 2022).

The HLT framework offers a structured yet flexible approach to teaching geometry. It allows for a progression of concepts aligned with students' cognitive development, moving from basic ideas like points and lines to advanced topics such as triangle congruence and geometric properties (Stein, Engel, & Smith, 2020). Teachers using HLT can scaffold learning by building on student's prior knowledge and adjusting instructional strategies based on continuous assessment. Research has demonstrated that HLT fosters active learning, critical thinking, and enhanced problem-solving skills. For instance, studies in the United States and Europe have shown that HLT improves students' ability to connect theoretical principles with practical applications, leading to deeper understanding and retention (Fennell, 2019; Natarajan et al., 2022). Implementing HLT could address systemic challenges in geometry education. The framework's emphasis on differentiation allows teachers to cater to the diverse learning needs of students, particularly in a country with significant disparities in educational resources. Students in rural areas, often disadvantaged by limited access to quality teaching materials, could benefit from a more inclusive and adaptive approach (Ogunyemi, 2021).

By incorporating hands-on activities and real-world applications, HLT can make abstract geometric concepts more tangible and engaging, improving both understanding and performance (Ajiboye, 2019). Moreover, integrating HLT into Nigeria's education system could enhance teacher professional development. The reflective practice required by HLT—through continuous assessment and strategy adaptation—aligns with calls for improved pedagogical training that promotes critical thinking, problem-solving, and student-centred learning (Ogunyemi, 2021). Teachers could also collaborate more effectively, sharing insights and best practices for implementing HLT in classrooms. This approach has the potential to not only improve geometry education but also foster a broader culture of innovation in teaching mathematics.

## 2. THE RESEARCH FOCUS

### Statement of the problem

The current state of geometry education faces significant challenges, as students often struggle to translate theoretical knowledge into practical application. Many can verbally explain how to construct geometric figures but fail to accurately execute these tasks due to insufficient hands-on experience. However, traditional teaching methods, such as abstract and overly theoretical instruction, further hinder understanding. These issues highlight the need for innovative approaches like the Hypothetical Learning Trajectory (HLT), which offers a structured and adaptive framework tailored to students' cognitive development. This study aims to improve student's learning outcomes and experiences in geometry by leveraging HLT to create more effective and engaging educational strategies.

### Purpose of the study

The purpose of this study is to examine the utilization of secondary school students' geometrical learning outcomes and experiences through hypothetical learning trajectories in Delta State. The specific objectives are:

1. To investigate how secondary school students progress through various stages of learning geometry using hypothetical learning trajectories.
2. To identify the difficulties encountered by secondary school students when learning geometry through hypothetical learning trajectories.
3. To explore the strategies employed by students to solve geometry problems within the framework of hypothetical learning trajectories.

### Research Questions

The following questions were raised

1. How do secondary school students progress through various stages of learning geometry using hypothetical learning trajectories?
2. What difficulties do secondary school students encounter when learning geometry through hypothetical learning trajectories?
3. What strategies do students employ to solve geometry problems within the framework of hypothetical learning trajectories?

## 3. RESEARCH METHODOLOGY

The study utilized a thematic research design, which is a qualitative approach focused on identifying and analyzing patterns or themes in the data (Braun & Clarke, 2006). This design enabled an in-depth exploration of how Hypothetical Learning Trajectories (HLTs) influenced students' experiences and outcomes in geometry. The study was conducted in Oshimili South Local Government Area (LGA) of Delta State, Nigeria, known for its diverse educational landscape and suitability for studying instructional methodologies.

The population consisted of about 3,026 senior secondary school students from 22 public junior secondary schools in the LGA, as recorded by the Local Education Authority. A simple random sampling technique was employed to select 8 schools from the 22 public schools in the LGA. From each selected school, 25 students were randomly sampled, resulting in a total sample size of 200 students. Data collection was carried out using a thematic approach, incorporating classroom observations and an analysis of students' work samples. These instruments captured rich qualitative data on how students engaged with HLTs and the progression of their geometrical understanding. The instruments were validated through face and content validity checks, involving two experts in mathematics education and research, who provided feedback to ensure comprehensive coverage of the study's constructs.

The method of data collection involved observing classroom dynamics and analyzing student work samples, including assignments and problem-solving tasks, to evaluate the effectiveness of HLTs in enhancing geometry learning. Data analysis was performed using thematic analysis, which involved transcribing, coding, and organizing data into categories and themes. The results were systematically structured in tables to highlight key insights, such as students' challenges, progress, and the impact of HLTs on their geometrical learning outcomes. This methodology provided a robust framework for understanding the role of HLTs in improving geometry education.

## 4. RESULTS

The results obtained were used in discussions and a summary of the findings.

### Data Presentation

**Research Question One:** How do secondary school students progress through various stages of learning geometry using hypothetical learning trajectories?

Table: Stages of Students' Progression in Learning Plane Geometry (Angles and Their Types, Properties of Lines and Angles) through Hypothetical Learning Trajectories

| Stage | Description | Student Examples/Observations | Theme |
|---|---|---|---|
| Initial Understanding | Students show limited or superficial understanding of geometric concepts such as angles and lines. | I only know basic shapes, but not much about the angles between them. | Surface-level understanding of concepts |
| Exploration and Identification | Students begin to recognize different types of angles (acute, obtuse, right, etc.) and explore the properties of lines. | I can see the different angles in triangles, but I don't know how to measure them. | Exploration and identification of properties |
| Application and Analysis | Students start applying their knowledge of angles and lines to solve basic geometry problems. | I was able to use the angle sum property of triangles to find missing angles in problems. | Application of geometric principles |
| Mastery and Problem Solving | Students demonstrate full understanding by solving complex problems involving properties of lines and angles. | Now, I can solve problems involving parallel lines and angles without help. | Mastery and problem-solving ability |

Table 1 outlines the progression of secondary school students in learning plane geometry, focusing on angles and their properties, through hypothetical learning trajectories (HLTs). It identifies four distinct stages of development. The first stage, Initial Understanding, reflects a superficial grasp of geometric concepts, where students recognize basic shapes but lack knowledge of angles and their relationships. The second stage, Exploration and Identification, shows students beginning to identify angle types (e.g., acute, obtuse, right) and exploring properties of lines, though they often struggle with tasks like measurement. In the third stage, Application and Analysis, students start applying their understanding to solve basic problems, such as calculating missing angles using the angle sum property of triangles. This stage marks the development of analytical skills.

Finally, the Mastery and Problem Solving stage demonstrates students' ability to independently solve complex problems involving geometric properties, showcasing comprehensive understanding and confidence. The table illustrates a structured progression in students' geometric learning, moving from basic recognition and exploration to advanced application and mastery. The trajectory emphasizes how conceptual understanding deepens with experience and practice, highlighting the role of targeted instruction in facilitating this growth.

**Research Question Two:** What difficulties do secondary school students encounter when learning geometry through hypothetical learning trajectories?

**Table 2: Difficulties Encountered by Secondary School Students When Learning Geometry through Hypothetical Learning Trajectories**

| Difficulty | Description | Student Examples/Observations | Theme |
|---|---|---|---|
| **Abstract Understanding of Concepts** | Students struggle to grasp abstract geometric concepts, especially when visualizing shapes and their properties. | I can't picture what a 3D shape looks like when I only have a 2D representation. | Difficulty in conceptualizing abstract ideas |
| **Translating Visual to Mathematical Language** | Students find it challenging to translate geometric visuals into formal mathematical language (e.g., angles, lines). | I don't know how to write down the equation for the angle I measured. | Struggles with language and notation |
| **Application of Theoretical Knowledge** | Students face difficulties in applying learned concepts to solve problems or analyze real-world situations. | I know the formula for angles, but I can't figure out how to use it in problems. | Difficulty in problem-solving and application |
| **Classroom Dynamics and Learning Environment** | Students experience distractions or limited opportunities for hands-on activities in geometry lessons. | It's hard to focus when there are so many students in the class. | Challenges in the learning environment |
| **Cognitive Overload** | Students feel overwhelmed when presented with too many concepts at once, making it difficult to retain and apply knowledge. | I get confused when we learn too many types of angles in one lesson. | Cognitive overload and information retention issues |

Table 2 reveals that students face significant challenges in learning geometry through hypothetical learning trajectories. These difficulties include grasping abstract concepts, translating visual representations into mathematical expressions, and applying theoretical knowledge in problem-solving. Also, classroom dynamics, such as large class sizes and distractions, hinder effective learning. Cognitive overload also poses a barrier, with students struggling to process multiple concepts introduced simultaneously. These findings emphasize the need for tailored teaching strategies, such as scaffolding and hands-on activities, to address these issues and enhance students' understanding and application of geometry.

**Research Question Three:** What strategies do students employ to solve geometry problems within the framework of hypothetical learning trajectories?

**Table 3: Strategies Employed by Students to Solve Geometry Problems within the Framework of Hypothetical Learning Trajectories**

| Strategy | Description | Student Examples/Observations | Theme |
|---|---|---|---|
| **Visualization** | Students use visual tools such as diagrams, drawings, and models to help solve geometry problems. | I draw the angles and lines on paper to see how they relate to each other. | Use of visual aids and tools |
| **Step-by-step Problem Solving** | Students break down complex geometry problems into manageable steps to simplify the process. | I start with the basics, like identifying the types of angles, and then move on to solving the problem. | Sequential approach to problem-solving |
| **Collaborative Learning** | Students work with peers to discuss and solve geometry problems, leveraging collaborative strategies. | I worked with my classmate to figure out how to apply the angle sum rule in triangles. | Peer collaboration and discussion |
| **Use of Mathematical Formulas** | Students apply known geometric formulas and properties to solve problems efficiently. | I always use the angle sum property of triangles to solve for unknown angles. | Application of geometric formulas and properties |
| **Trial and Error** | Students use a trial-and-error approach, testing different solutions until they find the correct one. | When I was unsure, I tried different angle measures until the sum was added up correctly. | Problem-solving through experimentation |

Table 3 highlights various strategies students employed to solve geometry problems within the framework of Hypothetical Learning Trajectories (HLTs). Visualization emerged as a prominent approach, with students using diagrams and drawings to conceptualize relationships between angles and lines. A step-by-step method was frequently utilized, simplifying complex problems into manageable components. Collaboration with peers also proved effective, fostering idea-sharing and problem-solving. Students demonstrated the practical application of geometric formulas and properties, while trial-and-error techniques allowed them to refine solutions. These strategies underscore the importance of diverse, student-centered approaches in enhancing geometrical problem-solving skills.

## 6. DISCUSSION OF FINDINGS

Table 1, explores how secondary school students progress through stages of learning geometry using hypothetical learning trajectories (HLTs), focusing on angles and their properties. Four stages were identified: Initial Understanding, where students show superficial knowledge; Exploration and Identification, where they recognize angle types but struggle with measurement; Application and Analysis, involving solving basic problems; and Mastery and Problem Solving, marked by solving complex problems independently. The findings align with Battista (2018) and Herzog Jensen, & Tzekaki, (2022), who affirm the progression stages and emphasize instructional strategies for deeper understanding. However, there are disagreements. For example, Jones and Tzekaki (2020) suggest scaffolding can accelerate learning, and Martin, Slavin, & Chen, (2021) argue that digital tools may lead to earlier mastery. Also, Nguyen and Taylor (2023) challenge the universality of "mastery," highlighting socio-cultural influences on learning.

Table 2, reveals complex challenges secondary school student's face in learning geometry through Hypothetical Learning Trajectories (HLTs). Abstraction difficulties and integrating visual-mathematical skills highlight the need for enhanced pedagogical approaches. Classroom dynamics and cognitive overload emphasize the importance of adaptive teaching methods, such as using visual aids and pacing lessons to match students' cognitive needs. The findings concur with Pratiwi Putri, & Sari, (2020) and Kuncoro Lestari, & Yuliana (2023), who emphasized the value of scaffolding and visual tools.

The findings in Table 3, suggest that students employ diverse strategies to solve geometry problems within the framework of Hypothetical Learning Trajectories (HLTs). Visualization, a key strategy, aligns with research by Kuncoro, et al(2023), who highlighted the role of visual aids in enhancing conceptual clarity. Step-by-step problem-solving, as observed, concurs with Pratiwi, Putri, & Sari,(2020), emphasizing structured approaches to handling geometric problems. Collaborative learning strategies resonate with Asomah, Osei, & Antwi, (2023), who noted that peer interactions improve understanding and problem-solving abilities. However, The use of trial-and-error reflects the adaptive problem-solving techniques discussed by Adams and Enu (2023), but they advocate for reducing dependency on this method through better scaffolding. Collectively, these findings reinforce the importance of integrating visualization, structured problem-solving, and collaborative approaches to strengthen students' geometrical learning experiences within HLT frameworks.

## 6. CONCLUSION

This study examined how secondary school students progress through learning geometry using Hypothetical Learning Trajectories (HLTs). The findings indicate that HLTs are effective in enhancing students' understanding of geometric concepts, promoting problem-solving confidence, and fostering engagement. Students actively employ strategies such as visualization, step-by-step problem-solving, and collaborative learning, all of which align with contemporary teaching practices. However, challenges like abstract understanding, cognitive overload, and limited resources highlight areas for improvement in the application of HLTs.

## 7. RECOMMENDATIONS

1. Teachers should integrate visual tools like diagrams and 3D models to help students understand abstract geometric concepts more clearly.
2. Students should be encouraged to work together in problem-solving exercises to enhance their learning experiences and foster peer-to-peer teaching.
3. Teachers should break down complex problems into manageable steps to help students better grasp challenging concepts,
4. Teachers should incorporate hands-on activities and real-world applications to make lessons more engaging and relevant to students' lives.
5. Teachers should use digital tools (like interactive software and geometry apps) to enhance the learning experience and cater to diverse learning styles. The aim is to make learning geometry more interactive, accessible, and effective.
6. Teachers should pace lessons according to students' cognitive abilities, gradually introducing more complex concepts to prevent overload and enhance retention.

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