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Thin Flame Effect on Thermal Explosion in the Interior of the Earth

[1]A.O. Popoola, M.O. Olayiwola & M.K. Kolawole
Department of Mathematical and Physical Sciences,
Faculty of Basic and Applied Sciences,
Osun State University
Osogbo Nigeria.

Corresponding author: amos.popoola@uniosun.edu.ng

ABSTRACT

Problem statement: The paper reviews the combustion processes that occur in the interior of the earth during Gravitational Differentiation GD and Radioactive Decay RD processes which are characterized by ignition and explosion. The paper therefore examine the effect of flame thickness on maximum temperature of the reaction during the processes. The partial differential equation governing the model together with the boundary conditions, is transformed into ordinary differential equation by thin flame technique. The resulting equation is investigated for the effects of some sensitive factors such as activation energies ratio, flame thickness, gravitational differentiation and radioactive decay on maximum temperature of the reaction which occurs in the earth interior. The criteria for the existence of unique solution of the resulting equations are established. Numerical results were obtained by shooting method. The results show that flame thickness, activation energies ratio, gravitational differentiation and radioactive decay have appreciable effects on maximum temperature of the reaction. In particular, regulating the flame thickness has helpful implication on the reactions in terms of heat release.

Keywords: Homogenous, thermal explosion, activation energy, Gravitational Differentiation (GD), Blow up, temperature, activation energy ratio, radioactive decay, exothermic.

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1. INTRODUCTION

Most of our contributions in combustion theory and modelling laid emphasis on heat release during the reaction [1-3]. This is basically required for safety and industrial purposes. Literatures have shown that in the interior of the earth, there are two major sources of heat that constitute exothermic regimes; These are Gravitational Differentiation (GD) and decay of radioactive elements [4]. Literatures further established that in the early stages of planetary build up, the earth was much less compact than what it is today. This build-up process led to more gravitational attractions which force the earth to contract into smaller volume. During the gravitational differentiation, the potential energy generated becomes heat energy due to viscous dissipation. Also, radioactive elements are inherently unstable; The unstable Uranium isotope (Uranium-238) slowly decay to Lead - 206 and the radioactive decay processes continue. They break down over time to more stable forms and release intense heat (and may include flame) as by-product of the chain reaction. This heat is continually radiated outward through several concentric shells that form the solid portion of the planet.

The work done by [4] explained the above processes in the earth interior and gave more understanding that thermal processes that occur in the earth's interior differ from characteristic thermal explosion but they are analogous. Other contributions by [5-7] also established that although multiple steps are involved in reaction but two major steps are basically involved. Those steps include chemical decomposition and combustion process. Our previous work on thermal explosion showed that explosion generally results from two exothermic reactions; one step follows the other in very rapid succession depending on the activation energies of the reactions [9]. [10] presented some remarks on thermal explosion in the early evolution of the earth. The paper considered the unsteady and steady state energy equation associated with the earth evolution, and establish the criteria for the occurrence of thermal runaway.

[11] also investigated the effect of radioactive heat source and gravitational differentiation on unsteady state thermal explosion in the evolution of the earth. The resulting energy equation was solved by shooting method. The authors showed that critical temperature which signifies the onset of thermal instability due to gravitational differentiation depends linearly on the intensity of radioactive heat source. We were able to show that even when the thermal conductivity due to gravitational differentiation and ordinary thermal conductivity are comparable, a steady thermal solution exists under specified conditions.

[12] revisited the theory of evolution of the earth. The effect of gravitational differentiation in the separation of heavier material forming the earth's core from Silicates in the extended and heated area was studied. Previous literatures focused on small and large thermal conductivities but we focused on all orders of thermal conductivity. The numerical solution of the energy equation was provided by shooting method. The previous results in the literatures were special cases of the new results in that paper. Our contributions on heat transfer were not restricted to [13-14] alone. [15] investigated the effect of activation energies on thermal explosion that occurs in the interior of the earth during gravitational differentiation and decay of radioactive substances. The unsteady, steady and homogeneous reactions were considered and studied. Theorems on the existence of unique solution were formulated and proved. Results on blow-up were obtained, and the criteria for a blow up to occur in the chain reaction, were established. The analytical and numerical results showed that activation energies have different implications in terms of heat release.

While our contributions on safety and modelling [16-18] continue, [19] also examined thermal explosion arising from time-dependent gravitational differentiation and radioactive decay in the Earth's Interior. He considered the work done by [15] when the gravitational differentiation and radioactive decay parameters are time dependent. The paper was able to establish that the problem has a unique solution and the activation energies ratio have appreciable effects on the reactions in terms of heat release. This paper therefore, considers [15, 18] and uses thin flame technique to reduce the partial differential equation into ordinary differential equation and to also introduce flame thickness parameter into the model in order to study its effect on the maximum heat release.

2. MATERIAL AND METHODS

Following [15,18], the dimensionless thermal conductivity equation governing the generation of heat by two major sources; GD and the decay of radioactive elements, is given by

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial \xi}\left(1 + P_{eo} e^{\frac{(1+\alpha)\theta}{n}}\right)\frac{\partial \theta}{\partial \xi} + \Gamma_d e^{(1+\alpha)\theta} + \Gamma_r \tag{1}$$

together with the initial and boundary conditions

$$\theta(\xi,0) = 0, \qquad \theta(-1,\tau) = \theta(1,\tau) = 0 \tag{2}$$

where the parameters are defined as;

θ	Non-dimensional temperature
τ	Non-dimensional time variable
ξ	Non dimensional space variable.
P_e	Péclet number
α	Ratio of activation energies
Γ_d	Non dimensional term for Gravitational Differentiation
Γ_r	Non dimensional term for radioactive source

The equation (1) subject to conditions (2), becomes

$$\frac{\partial \theta}{\partial \tau} = \left(1 + P_{eo} e^{\frac{(1+\alpha)\theta}{n}}\right)\frac{\partial^2 \theta}{\partial \xi^2} + \frac{(1+\alpha)}{n} P_{eo} e^{\frac{(1+\alpha)\theta}{n}} \left(\frac{\partial \theta}{\partial \xi}\right)^2 + \Gamma_d e^{(1+\alpha)\theta} + \Gamma_r \tag{3}$$

2.1 Thin Flame Technique.

Let $\theta(\xi,\tau) = g(\eta)$ *such that* $\eta = \xi - v\tau.$ (4)

where v is the flame thickness.

The equation (3) gives

$$\left(1 + P_{eo} e^{\frac{(1+\alpha)g}{n}}\right)\frac{d^2 g}{d\eta^2} + v\frac{dg}{d\eta} + \frac{(1+\alpha)}{n} P_{eo} e^{\frac{(1+\alpha)g}{n}} \left(\frac{dg}{d\eta}\right)^2 + \Gamma_d e^{(1+\alpha)g} + \Gamma_r = 0 \tag{5}$$

and the above equation is now subject to conditions

$$g(-1-v\tau) = 0, \qquad g(1-v\tau) = 0 \tag{6}$$

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Case 1: $P_{eo} e^{\frac{(1+\alpha)g}{n}} >> 1,$.

The equation (5) remains as

$$\left(1+P_{eo} e^{\frac{(1+\alpha)g}{n}}\right)\frac{d^2 g}{d\eta^2}+ v\frac{dg}{d\eta} + \frac{(1+\alpha)}{n} P_{eo} e^{\frac{(1+\alpha)g}{n}} \left(\frac{dg}{d\eta}\right)^2 + \Gamma_d\, e^{(1+\alpha)g} + \Gamma_r = 0 \qquad (7)$$

satisfying $g(-1-v\tau) = g(1-v\tau) = 0$ (8)

2.2 Existence of Unique Solution

Theorem 1: Let D denote the region for which $0 \le \alpha \le N$, $-1 \le y_1 \le 1$, $v, N, \Gamma_r, \Gamma_d, n, P_{eo} > 0$. *Then problem (7) which satisfies conditions (8) and for which* $g'(-1-v\tau)$ *is fixed, has a unique solution .*

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \eta \\ g \\ g' \end{pmatrix} \qquad (9)$$

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ y_3 \\ -\dfrac{\left[v y_3 + \dfrac{(1+\alpha) p_{e_0}}{n} \exp\left(\dfrac{(1+\alpha) y_2}{n}\right) (y_3)^2 + \Gamma_d \exp((1+\alpha)\, y_2) + \Gamma_r \right]}{\left(1 + p_{e_0} \exp\left(\dfrac{(1+\alpha) y_2}{n}\right)\right)} \end{pmatrix} \qquad (10)$$

$$= \begin{pmatrix} g_1(y_1, y_2, y_3) \\ g_2(y_1, y_2, y_3) \\ g_3(y_1, y_2, y_3) \end{pmatrix}$$

satisfying the initial conditions

$$\begin{pmatrix} y_1(-1-v\tau) \\ y_2(-1-v\tau) \\ y_3(-1-v\tau) \end{pmatrix} = \begin{pmatrix} -1-v\tau \\ 0 \\ -\lambda_g \end{pmatrix} \qquad (11)$$

Remark: λ_g is guessed such that the boundary condition $y_2(1-v\tau)=0$.

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Theorem 2: Let D denote the region for which $0 \le \alpha \le N, -1 \le y_1 \le 1$, $0 \le y_2 \le M$, $\lambda_g \le y_3 \le -\lambda_g^*$, $v, M, N, \Gamma_r, \Gamma_d, n, P_{eo} > 0$. *The functions* $g(i = 1, 2, 3)$ *are Lipschitz continuous in D.*

Proof:

Clearly, $\left|\frac{\partial g_i}{\partial y_j}\right|, i, j = 1,2,3$, are continuous in D and bounded on D. So, there exist a constant K such that $K = \left|\frac{\partial g_i}{\partial y_j}\right|, i, j = 1,2,3$. Hence $g_i(y_1, y_2, y_3), i = 1, 2, 3$ are Lipschitz continuous and so problem (10) satisfying (11) is Lipschitz continuous.

Proof of theorem 1: The existence of Lipschitz constant in the proof of theorem 2 implies the existence of unique solution of problem (10) which satisfies (11). And this implies the existence of unique solution of problem (6) satisfying the conditions (7).

Case 2: $P_{eo} e^{\frac{(1+\alpha)g}{n}} << 1,$

The equation (4) becomes

$$\frac{d^2 g}{d\eta^2} + v\frac{dg}{d\eta} + \Gamma_d e^{(1+\alpha)g} + \Gamma_r = 0 \tag{12}$$

satisfying the condition (8)

The equation (12) is resolved into a system of equations as follows;

Recall that
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} \eta \\ g \\ g' \end{pmatrix} \tag{13}$$

Then

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ y_3 \\ -\left[-\frac{y_1 y_3}{2} + \Gamma_d \exp((1+\alpha) y_2) + \Gamma_r\right] \end{pmatrix} \tag{14}$$

satisfying the initial conditions

$$\begin{pmatrix} y_1(-1-v\tau) \\ y_2(-1-v\tau) \\ y_3(-1-v\tau) \end{pmatrix} = \begin{pmatrix} -1-v\tau \\ 0 \\ -\lambda_z \end{pmatrix} \tag{15}$$

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3. NUMERICAL COMPUTATION

In this section, numerical solutions of problems (7) and (12) satisfying conditions (8) are provided by using Runge-Kutta Shooting method. Computer programmes written in Pascal language, were used to solve problems (10) and (14) together with conditions (11) and (15) respectively, and for which λ_g and λ_z are guessed such that the boundary condition $y_2(1-\nu\tau)=0$. The numerical results obtained are presented in the figures below.

<u>Case 1:</u> $P_{eo}e^{\frac{(1+\alpha)g}{n}} >> 1,$.

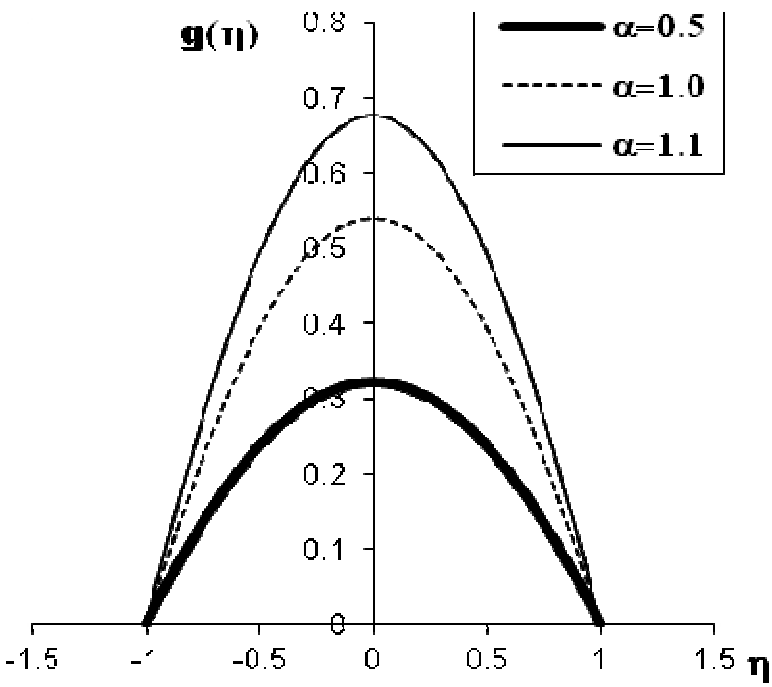

Figure 1: Temperature profile for fixed values of $\Gamma_r = 0.4$, $\Gamma_d = 0.2$, $\nu = 0.01$ and for various values of α.

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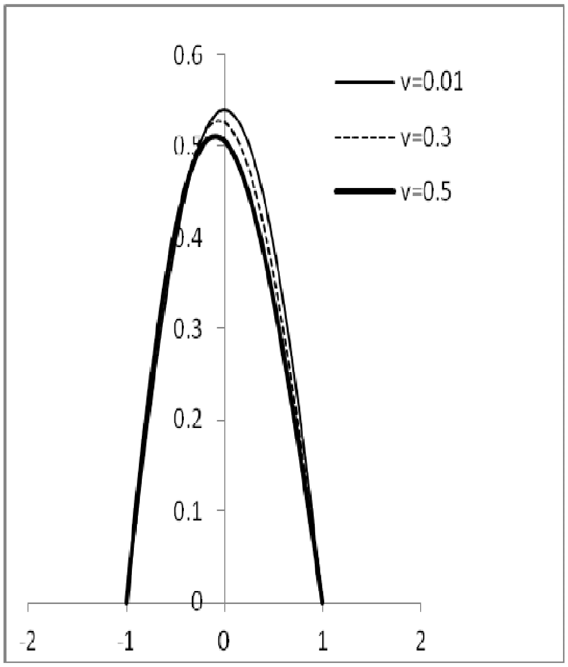

Figure 2: Temperature profile for fixed values of α=1, Γ_d= 0.2, Γ_r= 0.4, and for various values of v

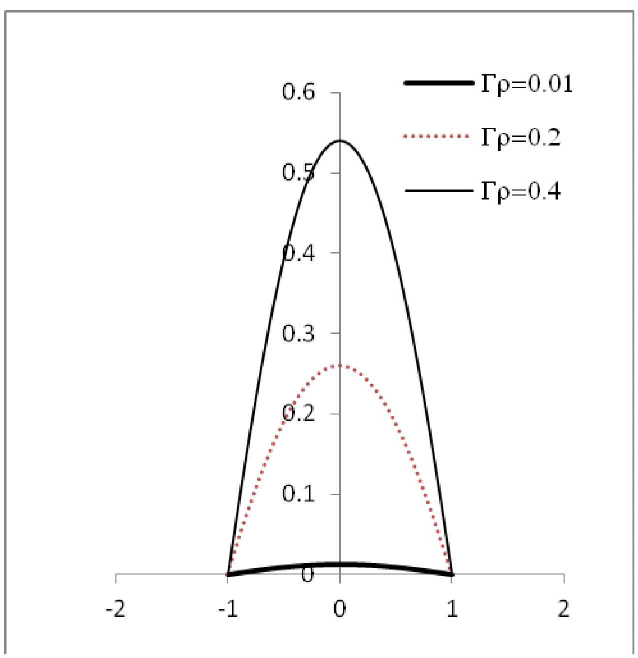

Figure 3: Temperature profile for fixed values of α=1, Γ_d= 0.2, v= 0.01 and for various values of Γr

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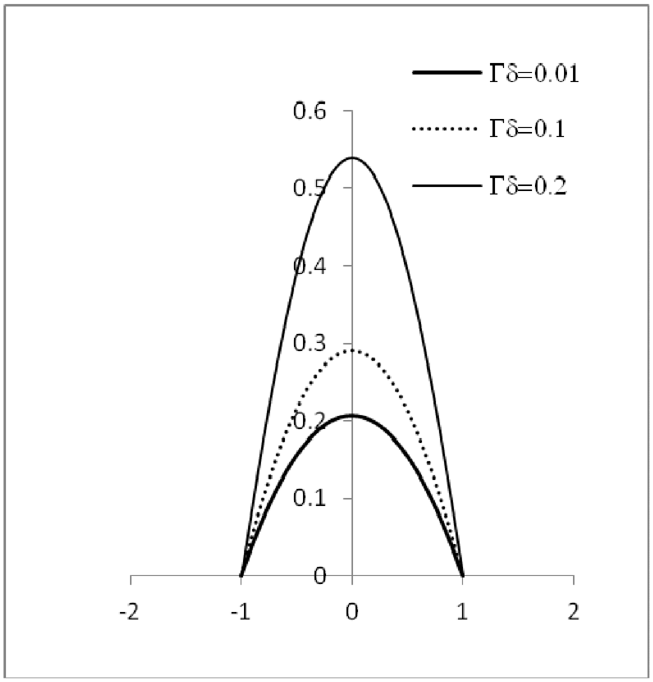

Figure 4: Temperature profile for fixed values of α=1, Γρ= 0.4, v= 0.01 and for various values of Γd

<u>Case 2:</u> $P_{eo} e^{\frac{(1+\alpha)g}{n}} << 1,$

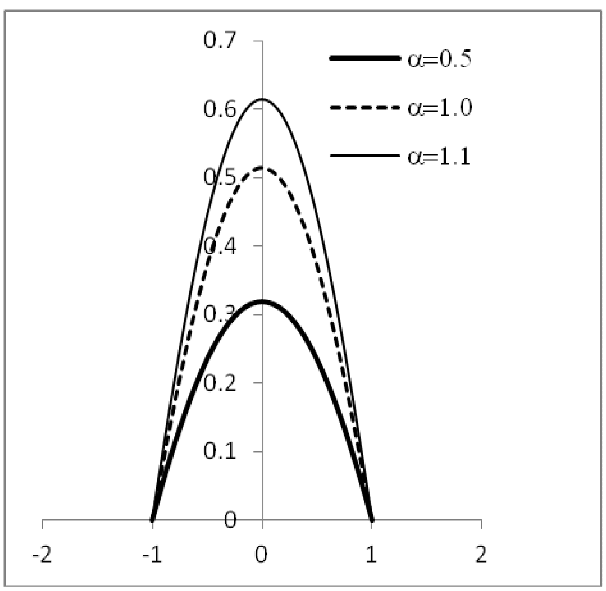

Figure 5: Temperature profile for fixed values of $\Gamma_r = 0.4$, Γ_d= 0.2, v= 0.01 and for various values of α

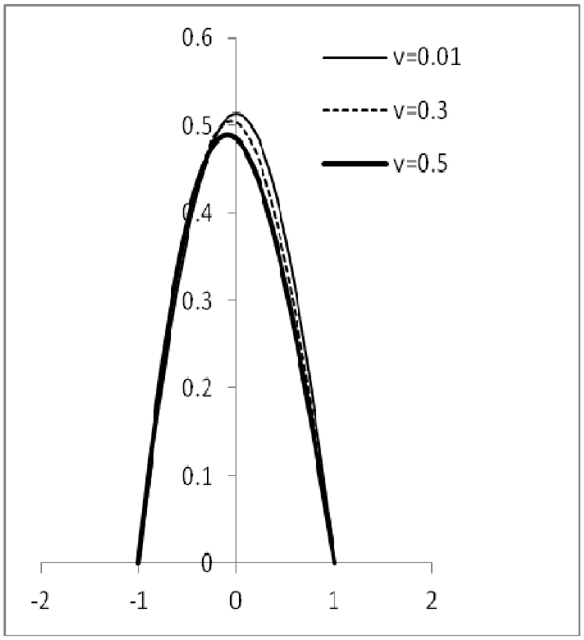

Figure 6: Temperature profile for fixed values of $\alpha=1$, $\Gamma_d = 0.2$, Gr= 0.4, and for various values of v

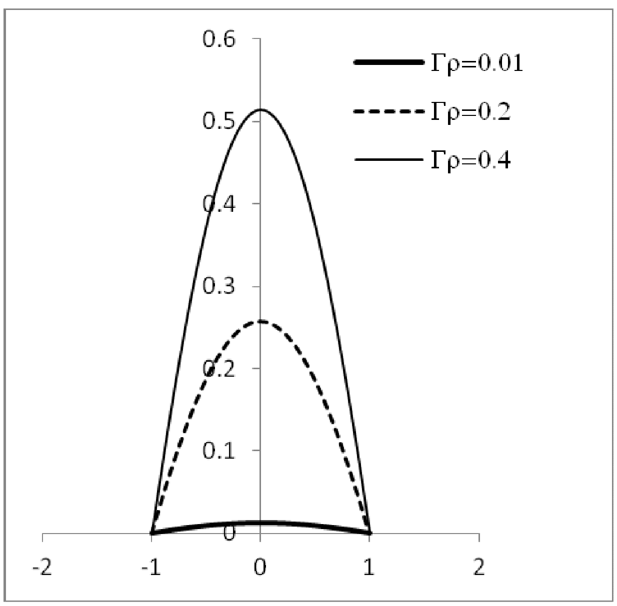

Figure 8: Temperature profile for fixed values of $\alpha=1$, $\Gamma\rho = 0.4$, v= 0.01 and for various values of Γd

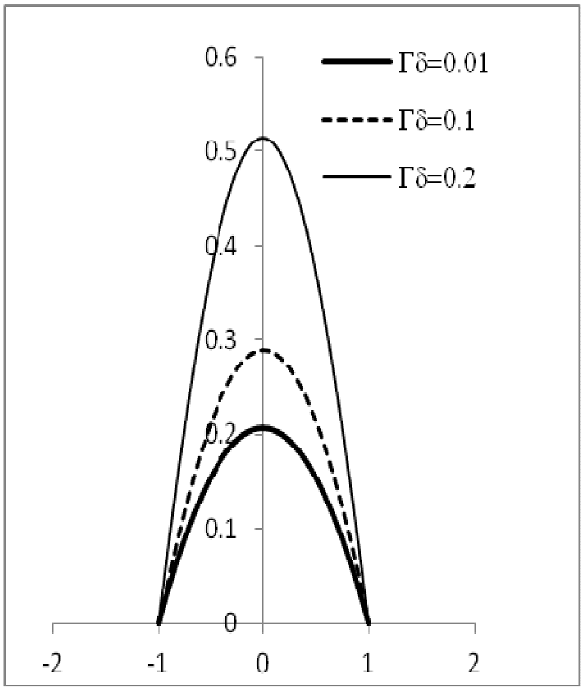

Figure 8: Temperature profile for fixed values of α=1, Γρ= 0.4, v= 0.01 and for various values of Γd

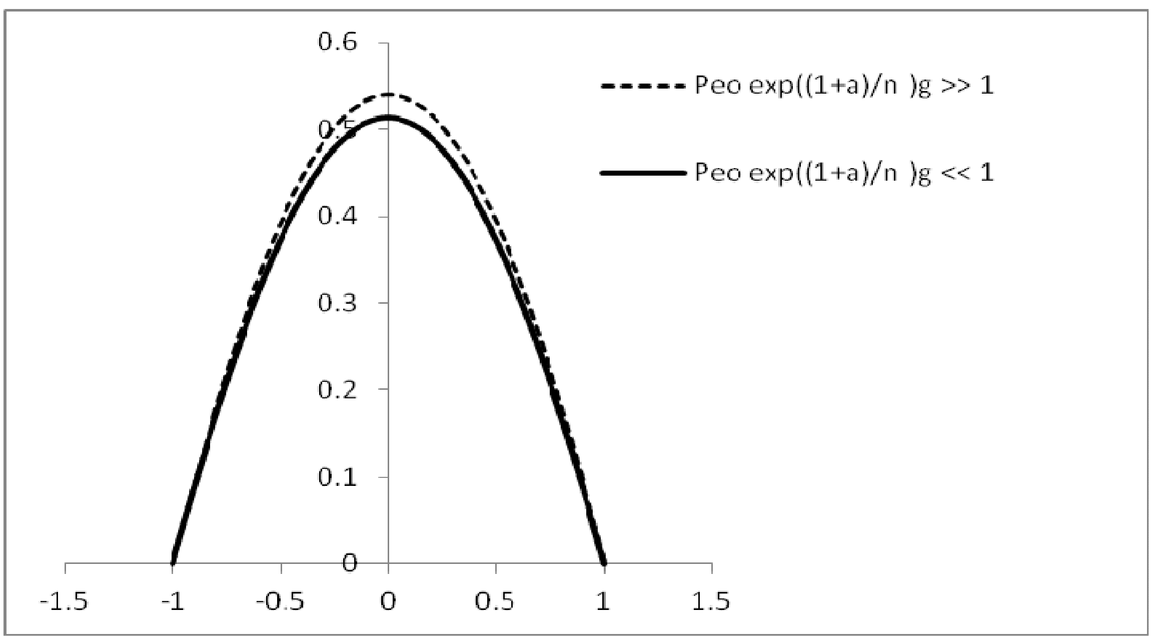

Figure 9: Temperature profile for fixed values of $\Gamma_r = 0.4$, Γ_d= 0.2, v= 0.01, α=1.0, showing cases 1 and 2

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4. RESULTS AND DISCUSSION

The paper examined the effects of flame thickness, and other sensitive factors such as activation energy ratio, gravitational differentiation and radioactive decay, on maximum temperature of the reaction during thermal explosion that may occur in the earth interior.

The model, was considered under the following cases;

(i). <u>Case 1</u>: $P_{eo}e^{\frac{(1+\alpha)g}{n}} >> 1$ (heat transfer due to GD or thermal convection far exceeds the conductive heat transfer.) Vityazev (2004), and

(ii). <u>Case 2</u>: $P_{eo}e^{\frac{(1+\alpha)g}{n}} << 1$ the contrast of (i).

Theorems establishing the criteria for the existence of unique solution of the resulting equations were formulated and proved. The proofs of theorems showed that the problems has a unique solution and the model therefore represents a physical problem. The resulting systems of equations were solved numerically by shooting method.

- Figures 1 and 5 showed that the activation energy ratio (□) has appreciable effects on maximum temperature of the reaction. As □ increases, the maximum temperature also rises.
- Figures 2 and 6 showed that the flame thickness (v) has appreciable effects on maximum temperature of the reaction. A reasonable adjustment of v produces maximum temperature of the reaction.
- Figures 3 and 7 showed that the heat release from the radioactive decay process ($□_r$) has appreciable effects on maximum temperature of the reaction. A rise in $□_r$ increases the maximum temperature of the reaction.
- Figures 4 and 8 showed that the heat release during Gravitational Differentiation ($□_d$) from has significant effects on maximum temperature of the reaction. A rise in $□_d$ increases the maximum temperature of the reaction.
- Figure 9 showed the difference between the two cases. The maximum temperature increases in case 1 when heat transfer due to GD or thermal convection far exceeds the conductive heat transfer.

5. CONCLUSION

In conclusion, flame thickness and other sensitive factors have appreciable effects on maximum temperature of the reactions during thermal explosion that may occur in the interior of the earth. In particular, regulating the flame thickness will be helpful in terms of heat release.

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