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A Computational Approach to Logistic Model using Adomian Decomposition Method.

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ABSTRACT

This research work examines a numerical approach to solving the Logistic Model. The paper also examines the applications of such a model in some related fields. The aim of this research paper is to compare the block integrator derived by Odekunle et al. with Adomian decomposition method (ADM) and see how they behave for solving Logistic Models. The results obtained is tabulated in order to compare the two results and the error given.

Keywords: ADM, Block Integrator, IVPs, Models, Logistic Equation

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1. INTRODUCTION

The Logistic Model was published by Verhulst, he derived his logistic model to describe the self-limiting growth of a biological population. Many animal species are fertile only for a brief period during the year and the young are born in a particular season so that by the time they are ready to eat solid food it will be plentiful. For this reason, the system might be better described by a discrete difference equation than a continuous differential equation. This is because not every existing animal will reproduce (a portion of them are male after all), not every female will be fertile, not every conception will be successful, and not every pregnancy will be successfully carried to term; the population increase will be some fraction of the present population. Therefore, if y_n is the number of animals this year and y_{n+1} is the number next year, then

$$y_{n+1} = \alpha\, y_n \qquad (1)$$

Where α (the growth rate) will approximate the evolution of the population. Logistic model produces exponential growth without limit (Elert, 2007). Since every population is bound by physical limitations of its surrounding, some allowance must be made to restrict this growth.

If there is a carrying capacity of the environment, then the population may not exceed that capacity. If it does, the population would become extinct. This can be modelled by multiplying the population by a number that approaches zero as the population approaches its limit. If we normalize y_n to this capacity, then the multiplier $(1-y_n)$ will suffice and the resulting logistic equation becomes,

$$y_{n+1} = \alpha\, y_n\, (1-y_n) \qquad (2)$$

We can also let N represents population size and t represents time, the logistic model is formalized by the differential equation,

$$\frac{dN}{dt} = \alpha N\left(1 - \frac{N}{K}\right) \qquad (3)$$

Where α remains the growth rate and k the carrying capacity. From (3), the early unimpeded growth rate is modelled by the first term αN, latter as the population grows; the second term $N^2\alpha/k$ which multiplied out becomes larger than the first as some members of the population N interfere with each other by competing for some critical resources, such as food or living space. This antagonistic effect is called *the bottleneck* and is modelled by the value of the parameter k . This competition diminishes the combined growth rate, until the value of N ceases to grow (this is called maturity of the population). Dividing both sides of (3) by k gives,

$$\frac{dN}{dtk} = \alpha N/k\left(1 - \frac{N}{K}\right) \qquad (4)$$

Now setting $y=N/k$ gives the differential equation,

$$\frac{dy}{dt} = \alpha y(1 - y) \qquad (5)$$

Substituting α=1 in (5) and imposing an initial condition $y(t_0)= y_0$ leads to the special logistic initial value problem,

$$y' = y(1-y),\ \ y(t_0) = y_0 \qquad (6)$$

It is important to note that (6) is of the form,

$$y'=f(x,y),\ y(a)= \eta \text{ for all } a \le x \le b \qquad (7)$$

Where f is continuous within the interval of integration $[a ,b]$. We assume that f satisfies Lipchitz condition which guarantees the existence and uniqueness of solution of (7).

Definition (Meyer, 1995)
A model is an object or concept that is used to represent something else. It is reality scaled down and converted to a form we can comprehend. Thus, a mathematical model is a model whose parts are mathematical concepts such as constants, variables, functions, inequalities, etc. Therefore, the attempt to describe some part of the real – world in mathematical terms is called mathematical modeling. It is an endeavour as old as antiquity but as modern as tomorrow's newspaper. That is why the logistic model finds application in virtually all fields of human endeavour. Ogunwale *et al.* (2010), in their paper also stressed that logistic model is an important class of non-linear regression model with great applications in growth and population studies. Mathematical techniques now play an important role in planning, managerial decision-making and economics which have probably been the longest quantified of the social sciences (Burghes *et al.*, 1980).

2. APPLICATIONS OF THE LOGISTIC MODELS

The Logistic Model finds applications in various fields, among which are;

i. **Neural Networks:** Logistic Models are often used in neural networks to introduce nonlinearity in the model and or to clamp signals within a specific range. A popular neural net element computes a linear combination of its input signals, and applies a bounded logistic function to the result; this model can be seen as a smoothed variant of the classical threshold neuron.

ii. **Statistics**: logistic functions are used in several roles in statistics; firstly, they are the cumulative distribution function of the logistic family of distribution. Secondly, they are used in logistic regression to model how the probability p of an event may be affected by one or more explanatory variables, an example would be to have the model,

$$p = P(+bx) \qquad (8)$$

iii. **Chemistry**: the concentration of reactants and products in autocatalytic reactions follows the logistic function.

iv. **Physics:** it is applied in Fermi distribution in the sense that the logistic function determines the statistical distribution of fermions over the energy states of a system in thermal equilibrium. In particular, it is the distribution of the probabilities that each possible energy level is occupied by fermions, according to FermiDirac statistics.

v. **Linguistics:** in linguistic, the logistic function can be used to model language change, an innovation that was at first marginal but has now become more universally adopted.

vi. **Economics:** the logistic function can be used to illustrate the progress of the diffusion of an innovation, infrastructures and energy source substitutions and the role of work in the economy as well as with the long economic cycle.

3. ADOMIAN DECOMPOSITION METHOD

The Adomian decomposition method, proposed by Adomian initially with the aims to solve frontier physical problem, has been applied to a wide class of deterministic and stochastic problems, linear and nonlinear, in physics, biology and chemical reactions etc. For nonlinear models, the method has shown reliable results in supplying analytical approximation that converges very rapidly. It is well known that the key of the method is to decompose the nonlinear term in the equations into a peculiar series of polynomials $\sum_{n=1}^{\infty} A_n$, where A_n are the so-called Adomian polynomials. Adomian formally introduced formulas that can generate Adomian polynomials for all forms of nonlinearity. Recently, a great deal of interests has been focused to develop a practical method for the calculation of Adomian polynomials An. However, the methods developed by R. Rach (1984), V. Seng et al. (1996), K. Abbaoui et al. (1994), S. Guellal et al.(1994), A.M. Wazwaz (2000), Y. Cherruault et al. (1992) and K. Abbaoui et al. (1995) also require a huge size of calculations. Wenhai Chen et al. (2004) established a promising algorithm that can be easily programmed in Maple, and be used to calculate Adomian polynomials for nonlinear terms in the differential equations.

Let us first recall the basic principles of the Adomian decomposition methods for solving differential equations. Consider the general equation Fu ¼ g, where F represents a general nonlinear differential operator involving both linear and nonlinear terms, the linear term is decomposed into L þ R, where L is easily invertible and R is the remainder of the linear operator.

For convenience, L may be taken as the highest order derivate. Thus the equation may be written as

Lu+ Ru+ Nu= g (9)

Where Nu represents the nonlinear terms. Solving Lu from (9), we have

Lu = g – Ru- Nu

Because L is invertible, the equivalent expression is

L^{-1}Lu = L^{-1}g – L^{-1}Ru- L^{-1}Nu (10)

If L is a second order operator, for example, L^{-1} is a twofold integration operator and L^{-1}Lu= u- u(0) - tu^{i}(0), then (10) for u yields

u = a+ bt+ $L^{-1}g - L^{-1}Ru - L^{-1}Nu$ (11)

Therefore, u can be presented as a series

u= $\sum_{n\to 0}^{\infty} u_n$ (12)

With u_o identified as a + bt + $L^{-1}g$ and u_n $(n > 0)$ is to be determined. The nonlinear term Nu will be decomposed by the infinite series of Adomian polynomials

$U_n = \sum_{n-0}^{\infty} A_n$ (13)

Where **An ,s** are obtained by writing

$v(\lambda) = \sum_{n=0}^{\infty} \lambda^n U_n$ (14)

$N(v(\lambda)) = \sum_{n=0}^{\infty} \lambda^n A_n$ (15)

Here λ is a parameter introduced for convenience. From (14) and (15), we deduce

$A_n = \frac{1}{n!}[\frac{d^n}{d\lambda^n} N(v(\lambda))]_{\lambda-0, n=0, 1,}$ ….. (16)

Now, substituting (12) and (13) into (11), we obtain

$$\sum_{n-0}^{\infty} Un = U_0 - L^{-1}R\sum_{n-0}^{\infty} Un - L^{-1}\sum_{n-0}^{\infty} An$$

Consequently, we can write

$u_o = a + bt + L^{-1}g$

$u_1 = -L^{-1}Ru_0 - L^{-1}A_{0,}$

K. I. Oshinubi et al. (2017) used the Adomian algorithm to solve autonomous and non-autonomous differential equation.

4. BLOCK INTEGRATOR

Collocation and interpolation procedures were used by choosing interpolation point s at a grid point and collocation points r at all points giving rise to $\varepsilon = s + r - 1$ system of equations whose coefficients are determined by using appropriate procedures. The approximation solution to (7) is taken to be a combination of power series and exponential function given by,

$$y(x) = \sum_{j=0}^{4} a_j x^j + a_5 \sum_{j=0}^{5} \frac{\alpha^j x^j}{j!} \qquad (17)$$

With the first derivative given by,

$$y(x) = \sum_{j=0}^{4} j a_j x^{j-1} + a_5 \sum_{j=1}^{5} \frac{\alpha^j x^j}{(j-1)!} \qquad (18)$$

Block integrators for solving (7) have initially been proposed by Milne (1953) who used them as starting values for predictor-corrector algorithm, Rosser (1967) developed Milne's method in form of implicit integrators, and Shampine and Watts (1969) also contributed greatly to the development and application of block integrators. More recently, authors like Butcher (2003), Zarina et al. (2005), Awoyemi et al. (2007), Yahaya et al. (2010), Areo et al. (2011), Badmus et al. (2011), Ibijola et al. (2011), Odekunle et al. (2012A) , Chollom et al. (2012), Odekunle et al. (2012B) have all proposed LMMs to generate numerical solution to (7). These authors proposed integrators in which the approximate solution ranges from power series, Chebychev's, Lagrange's and Laguerre's polynomials. The advantages of LMMs over single step methods have been extensively discussed by Awoyemi (2001).

5. NUMERICAL IMPLEMENTATION

We shall now proceed to implement the ADM algorithm on a special case of logistic Model. The implementation is carried out using *Maple* application language.

Problem

Consider the Logistic Model,

$$y^i = y(1-y),\ y(0) = 0.5 \qquad (19)$$

With the theoretical solution.

$$y(t) = \frac{0.5}{(0.5+0.5e^{-t})}$$

Using the algorithm derived by Wenhai Chen et al. (2004) equation (19) becomes

$$ICs := [0, [0.5]] : Eqs := [[u \rightarrow u, u \rightarrow -u^2, t \rightarrow 0]] : n := 5 : result := Adomian(ICs, Eqs, n);$$

$$result := [0.5 - 0.2500000000\,t + 0.02083333333\,t^3 - 0.002083333334\,t^5]$$

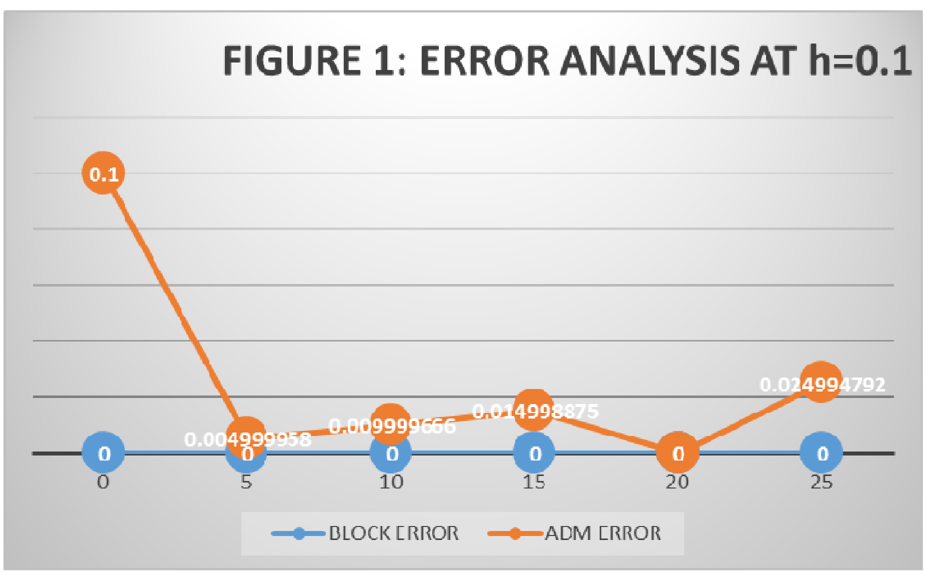
The above result was used to perform different iteration at h=0.1 and h=0.01.
The results are stated in the tables below

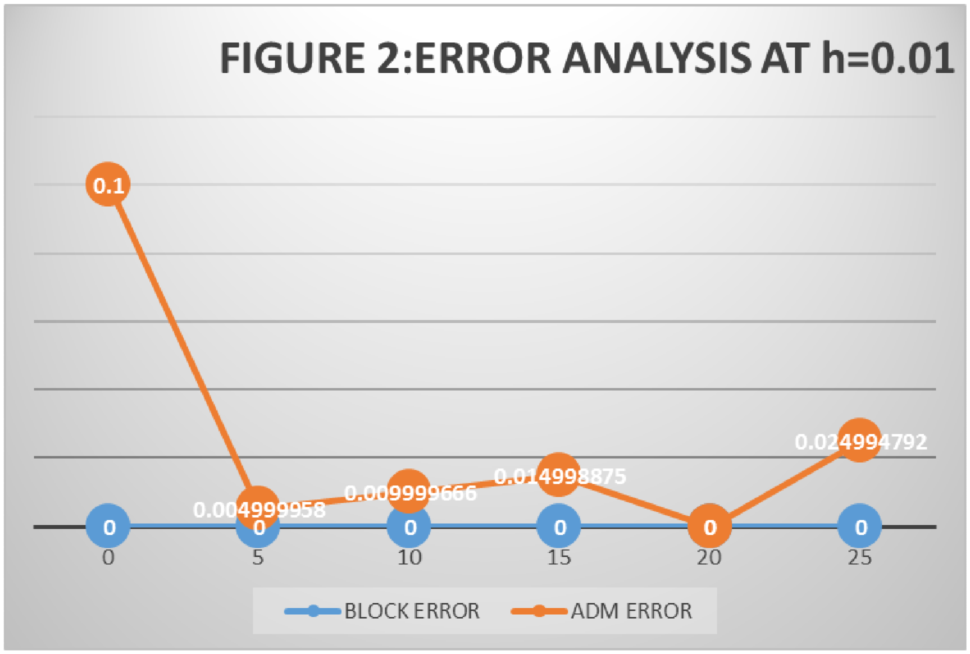
Table 1: ADM algorithm on equation (19) with h=0.1

t	Exact Solution	Block Integration (Odekunle et.al.)	ADM Solution	Block Integrator Error	ADM Error
0.0000000000	0.5000000000	0.5000000000	0.5000000000	0.0000000000	0.0000000000
0.1000000000	0.5249791874	0.5249791908	0.4750208125	-0.0000000034	0.0499583749
0.2000000000	0.5498339973	0.5498339991	0.4501660000	-0.0000000018	0,0996679973
0.3000000000	0.5744425168	0.5744425201	0.4255574373	-0.0000000033	0.1488850795
0.4000000000	0.5986876601	0.5986876562	0.4013120000	0.0000000039	0.1973756601
0.5000000000	0.6224593312	0.6224593350	0.3775390625	-0.0000000038	0.2449202687
0.6000000000	0.6456563062	0.6456530704	0.3543380000	0.0000032358	0.0291318306
0.7000000000	0.6681877721	0.6681877759	0.3317956875	-0.0000000038	0.3363920846
0.8000000000	0.6899744811	0.6899744751	0.3099840000	0.0000000006	0.3799904811
0.9000000000	0.7109495026	0.7109495023	0.2889573125	0.0000000003	0.4219921901
1.0000000000	0.7310585786	0.7310585764	0.2687500000	0.0000000022	0.4623085786

Table 2: ADM algorithm on equation (19) with h=0.01

t	Exact Solution	Block integration (Odekunle et. al.)	ADM Solution	Block Integrator Error	ADM Error
0.0000000000	0.5000000000	0.5000000000	0.4000000000	0.0000000000	0.100000000
0.0100000000	0.5024999791	0.5024999791	0.4975000208	0.0000000000	0.004999958
0.0200000000	0.5049998333	0.5049998333	0.4950001667	0.0000000000	0.009999666
0.0300000000	0.5074994375	0.5074994375	0.4925005624	0.0000000000	0.014998875
0.0400000000	0.5099986668	0.5099986668	0.4900013331	0.0000000000	0,019997333
0.0500000000	0.5124973964	0.5124973964	0.4875026035	0.0000000000	0.024994792
0.0600000000	0.5149955016	0.5149955016	0.4850044984	0.0000000000	0.029991003
0.0700000000	0.5174928576	0.5174928576	0.4825071423	0.0000000000	0,037389258
0.0800000000	0.5199893401	0.5199894015	0.4800106599	-0.0000000061	0.039978680
0.0900000000	0.5224848247	0.5224848247	0.4775151752	0.0000000000	0.044969649
1.0000000000	0.5249791874	0.5249791874	0.4750208125	0.0000000000	0.049958374





6. CONCLUSION

It was discovered that as we increased the step length the values we got using Adomian decomposition decreases. It was also observed that Adomian decomposition method error is large compare to the Odekunle et. al. Integrator block method.

It shows that Odekunle et. al. method is more accurate.

REFERENCES

[1] D. N. Burghesand A. D. Wood, "Mathematical models in the Social, Management and Life Sciences", Ellis Harwood Limited, 1980.
[2] G. Elert, "Measuring Chaos", The Chaos Hypertext book, 2007.
[3] W. J. Meyer, "Concepts of Mathematical Modeling", Mc Grow–Hill Book Company, 1995.
[4] R. E. Mickens, "Nonstandard Finite Difference Models of Differential Equations", World Scientific, Singapore, 1994.
[5] O. D. Ogunwale, O. Y. Halid, and J. Sunday, (2010), "On an Alternative Method of Estimation of
[6] PolynomialRegression Models",Australian Journal of Basic and Applied Sciences,vol. 4, no. 8, pp. 3585-3590, 2010.
[7] E. A. Areo, R. A. Ademiluyi and P. O. Babatola, "Three Steps Hybrid LinearMultistep Method for the Solution of First-Order Initial Value Problems in Ordinary Differential Equations",Journal of Mathematical Physics,vol. 19, pp. 261-266, 2011.
[8] D. O. Awoyemi, R. A. Ademiluyi and W. Amuseghan, "Off-grids Exploitation in the Development of More Accurate Method for the Solution of ODEs", Journal of Mathematical Physics, vol. 12, pp. 379-386, 2011.
[9] A.M. Badmus, D. W. Mishelia, "Some Uniform Order Block Method for the Solution of First-Order Ordinary Differential Equations", Journal of Nigerian Association of Mathematical Physics, vol. 19, pp. 149-154, 2011.

[10] J. C. Butcher, "Numerical Methods for Ordinary Differential Equations",West Sussex: John Wiley & Sons Ltd, 2003.
[11] J. P. Chollom, I. O. Olatunbosun, andS. Omagu, "A Class of A-Stable Block Explicit Methods for the Solution of Ordinary Differential Equations", Research Journal of Mathematics and Statistics, vol. 4, no. 2,pp. 52-56, 2012.
[12] G. G. Dahlquist, "Convergence and Stability in the Numerical Integration of Ordinary Differential Equations", Math. Scand.,vol. 4, pp. 33-50, 1956.
[13] E. A. Ibijola, Y. Skwame and G. Kumleng, "Formulation of Hybrid Method of Higher Step-sizes Through the Continuous Multistep Collocation",American Journal of Scientific and Industrial Research, vol. 2, no. 2, pp. 161-173, 2011.
[14] W. E. Milne, "Numerical Solution of Differential Equations", New York, Willey, 1953.
[15] M. R. Odekunle, A. O. Adesanya, and J. Sunday, "A New Block Integrator for the Solution of Initial Value Problems of FirstOrder Ordinary Differential Equations", International Journal of Pure and Applied Science and Technology,vol. 11, no. 1,pp. 92100, 2012A.
[16] M. R. Odekunle, A. O. Adesanya, and J. Sunday, "4-Point Block Method for the Direct Integration of First-Order Ordinary Differential Equations", International Journal of Engineering Research and Applications, vol. 2, no. 5,pp. 1182-1187, 2012B.
[17] J. B. Rosser,"A Runge-Kutta Method for all Seasons", SIAM Review, vol. 9, pp. 417-452, 1967.
[18] L. F. Shampine and H. A. Watts, "Block Implicit One-Step Methods",Mathematics of Computation, vol. 23, no. 108,pp. 731-740, 1969.
[19] Y. A. Yahaya andU. Mohammed, "Fully Implicit Three Point Block Backward Differentiation Formulae for Solution of FirstOrder IVPs",Leonardo Journal of Sciences,vol. 16, no. 1,pp. 21-30, 2010.
[20] B. I. Zarina, S. Mohammed, I. Kharil, and M. Zanariah, "Block Method for Generalized Multistep Adams Method and Backward Differentiation Formula in Solving First-Order ODEs", Mathematika, pp. 25-33, 2005.
[21] R. Rach, A convenient computational form for the Adomian polynomials, J. Math. Anal. Appl. 102 (1984) 415–419.
[22] V. Seng, K. Abbaoui, Y. Cherruault, Adomian polynomials for nonlinear operators, Math. Comput. Modell. 24 (1996) 59–65.
[23] K. Abbaoui, Y. Cherruault, Convergence of Adomians method applied to differential equations, Comput. Math. Appl. 28 (5) (1994) 103–109.
[24] S. Guellal, Y. Cherruault, Practical formulae for calculation of Adomians polynomials and application to the convergence of the decomposition method, Int. J. Biomed. Comput. 36 (1994) 223–228.
[25] A.M. Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators, Appl. Math. Comput. 111 (2000) 53–69.
[26] Y. Cherruault, G. Saccomandi, B. Some, New results for convergence of Adomians method applied to integral equations, Math. Comput. Modell. 16 (2) (1992) 85–93.
[27] K. Abbaoui, Y. Cherruault, New ideas for proving convergence of decomposition methods, Comput. Math. Appl. 29 (7) (1995) 103–108.
[28] C. Wenhai, L. Zhengyi, An algorithm for Adomian decomposition, Applied Mathematics and Computation 159 (2004) 221–235.
[29] J. Sunday, A. A. James, E. A. Ibijola3, R. B. Ogunrinde, S. N. Ogunyebi, A Computational Approach to Verhulst-Pearl Model**,** IOSR Journal of Mathematics. Volume 4, Issue 3 (Nov. – Dec. 2012), PP 06-13
[30] K.I Oshinubi and R.B Ogunrinde. An Adomian Decomposition Method for Autonomous and Non-Autonomous Ordinary Differential Equations, Asian Journal of Mathematics and Computer Research. Vol 19, Issue 2 (2017) 65-74