

# Convective Boundary Layer flow over an Exponentially Stretching Porous Surface with suction and injection

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## ABSTRACT

This study was conducted to investigate natural convective boundary layer flow, heat and mass transfer of incompressible viscous fluid over a porous stretching vertical surface in the presence of thermal radiation. A similarity transformation is used to reduce the governing system of PDEs to a set of nonlinear ordinary differential equations which are solved numerically using the fourth order Runge-kutta method with shooting technique. The validation of the present result was compared with previous studies and they compared favourably with minimal errors. The numerical computations were presented in tabular and graphical forms for various fluid parameters controlling the fluid flow, heat and mass transfer. It shows that an increase in radiation produced a rise on the velocity, temperature and concentration profiles. Skin friction, Nusselt and Sherwood numbers increased with increasing radiation. Skin friction, Nusselt and Sherwood numbers increase as convective terms that is Solutal and thermal Grashof parameters increase. There is need to invest on energy generation, dynamics of engineering and tap the significance of heat and mass transfer to improve and sustain our nation's dwindling economy.

**Keywords:** Convective Boundary, Layer, Flow, Porous Stretching Surface, Suction and Injection.

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### Aims Research Journal Reference Format:

Amoo, S.A. & Idowu, A.S. (2016): Convective Boundary Layer flow over an Exponentially Porous Stretching Surface with suction and injection. *Advances in Multidisciplinary Research Journal*. Vol 2, No. 1 Pp 19-28.

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## 1. INTRODUCTION

The study of convective boundary layer flow remains an application within the core engineering education. Advances in media available for the delivery of such research reports are challenges to the academics. Why the challenges? The challenges arise in as much as the government of day does not make use of such research reports from academics except the researches commissioned by such government in power. The importance and significance of such transport or flow over an exponentially porous stretching surface continue to agitate further research due to the enormous usage in industries that deal in electronic equipment. In fluid, the application areas of heat transfer in the human body, air conditioning system, electronic equipment, power plant, refrigeration systems too numerous to list. In the same vein, convective boundary-layer problems or flows are often controlled by injecting or withdrawing fluid through a porous bounding heated surface. This can lead to enhanced heating or cooling of systems and can delay the transition from laminar to turbulent flow.

Also, the movement of ship is always against the velocity on the high sea, the flows of aero plane in the case of turbulence in the air when the plane is airborne are all within the application of this flow. The ship movement, aircraft flows are to benefit from this research report the study like this will benefit the populace having regard for the effect of blowing and suction on free convection boundary layers; which are usually confined to cases with prescribed wall temperature. Ali, Zaman, Abidin, Naeemullah and Shah (2015) presented analytical solution for fluid flow over an exponentially porous stretching sheet in porous media. The authors used homotopy analysis method (HAM), the result obtained by HAM was compared with the numerical results presented in literature. There was a close agreement of the results with earlier results. Then the method also controlled the convergence of solution. Other previous works on the effects of blowing and suction on free convection boundary layers had been seen to confine to cases with prescribed wall temperature Hussain and Ahmad (2015), Idowu and Amoo (2015). In the past, the study of hydrodynamic flow and heat transfer over a porous stretching sheet has attracted considerable attentions due to its vast applications in many manufacturing processes Mukhopadhyay (2012).

This means it has important bearings on several technological and natural processes too numerous to catalogue. The production of sheet materials arise in a number of industrial manufacturing processes that include both metal and polymer sheets. Researchers have noted that the flow in a boundary layer separates in the regions of adverse pressure gradient and the occurrence of separation several undesirable effects in so far as it leads to increase in the drag on the body immersed in the flow and the adversely affected the heat transfers from the surface of the body. Several methods have been developed for the purpose of artificial control of flow separation. Separation can be prevented by suction as the low-energy fluid in the boundary layer is removed Amoo and Idowu (2015a, 2015b). On the contrary, the wall shear stress and hence the friction drag is reduced by blowing. Hussain and Ahmad (2015) investigated the effect of radiation on free convection flow of fluid with variable viscosity from a porous vertical plate. By this, it also suggests that free convective phenomenon has been the object of extensive research. The importance of this phenomenon is increasing day by day due to the enhanced concern in science and technology about buoyancy induced motions in the atmosphere, the bodies in water and quasi-solid bodies such as earth. Natural convection flows driven by temperature differences are very much interesting in case of Industrial applications. Buoyancy plays an important role where the temperature differences between land and air give rise to a complicated flow and in enclosures such as ventilated and heated rooms (Elbashbeshy (2001), Dada and Disu (2015)).

The radiation effects have important applications in physics and engineering particularly in space technology and high temperature processes. However, very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids. An investigation was carried out on the numerical solution for thermal radiation effect on inclined magnetic field of MHD free convective heat transfer dissipative fluid flow past a moving vertical porous plate with variable suction by Amoo and Idowu (2015b). Dada and Disu (2015) investigated heat transfer with radiation and temperature dependent heat source in MHD free convection flow in a porous medium between two vertical wavy walls. In the same vein, Uwanta (2011) investigated mass transfer of free convective flow over a vertical plate with heat sunk and jumped wall temperature; found out that for a fixed time, with increasing Schmidt number, indicative that concentration profiles are reducing while for fixed Schmidt number and increasing time, the concentration profiles are increasing. The reported that temperature reducing with increasing Prandtl number. Bello (2012) investigated the influence of dissipative friction and variable viscosity on unsteady hydro magnetic flow of a radiating gas inside a vertical porous channel. The finding revealed that increase in viscous dissipative friction cause a sharp increase in viscous the induced magnetic field flow rate, temperature and velocity in the absence of variable fluid properties.

The effect of dissipative friction was highly felt on the variable viscosity property on the flow. Abah *et al.* (2014) present the effect of mass and radiative heat transfer on free convective flow of a viscous incompressible optically thick fluid towards a vertical surface. The non-linear, non-dimensional similarity, transformed boundary-layer equations governing the problem are solved using fourth-order Runge-Kutta integration scheme and shooting iteration technique, the analysis shows that the radiation parameter  $N$  increases as temperature decreases; an increase in the Prandtl number leads to a decrease in the temperature profile, a rise in the thermal Grashof and mass transfer number leads to increase in the velocity profile and a rise in the Schmidt number leads to decrease in the concentration profile. Mukhopadhyay and Layek (2008) reported on free convective boundary layer flow and heat transfer of a fluid with variable viscosity on porous stretching surface in presence of thermal radiation. Fluid viscosity in their study is assumed to vary as a linear function of temperature. This was implemented using Lie Method for numerical analysis. Amoo (2015) presented effects of thermal radiation on heat and mass transfer over an exponentially stretching porous surface. The researcher added solutant parameter with concentration. The numerical analysis used was implemented using Shooting techniques with Runge-Kutta method. Also, Elsayed *et al.* (2014) investigated mass transfer over unsteady stretching surface embedded in porous medium in the presence of variable chemical reactions and suction/injection, the results show that the velocity and concentration profile decreases with increase in permeability parameter, suction/injection parameter and concentration decrease and unsteadiness. The chemical reaction parameter, while the velocity has negligible change especially at low rate of chemical reaction.

The heat transfer is a science that determines how and what rate heat energy is transferred. As a result of a temperature gradient or differences, Heat transfer occurs in three modes Conduction, Convection and Radiation. In Conduction, heat transfers from the region of higher temperature to a region of lower temperature by kinetic motion or direct impact of the molecules whether the medium is at rest or in motion. Convection is a mechanism in which the heat transfers because of the movement of fluid from one region to the other region in the medium. The conversion of the internal energy of a substance into a radiant energy is referred to as radiation heat transfer. Free or Natural Convection flow is caused by natural forces such as buoyancy forces which arise from density differences in a fluid. These density differences are the consequence of temperature and concentration gradients within the fluid. In mixed convection the order magnitude of the buoyancy force is comparable to the externally maintained pressure drop to force the flow. In literature, Elbashbeshy, 2001; Ishak, 2011; Srivas and Kishan, 2015; Sulochana and Kumar, 2015) have tried to present various outcomes of their studies to support the importance of convective fluid flows.

In order to estimate the heat transfer rate in the medium, we need to determine the distribution of the temperature field. The temperature field is determined by solving the heat equation, which is a statement of conservation of energy or the first law of thermodynamics. The term mass transfer means the tendency of a component in a mixture to travel from a region of high concentration to a region of low concentration. There are two basic modes of mass transfer: Diffusion and Convection. There is a close similarity between heat transfer and mass transfer in terms of the transport rate equation and transport conversion equation (Bidin and Nazar, 2009; Bhattachryya, 2011 and 2012). The convection mass transfer is analogous to convection heat transfer and occurs between a moving mixture fluid species and an exposed solid surface. Like heat transfer rates, the species mass flux can be determined from the mass conservation field by solving the species mass conservation equation which is a statement of conservation of mass species.

A number of analytical solutions exist when solving modelled problems in fluid dynamics but difficult to achieve in most cases. Where the exact solutions cannot be achieved the need for approximate solution in form of numerical analysis come to play and this gives room for comparisons among the researchers, and the last but not the least after numerical analysis is experimental. Several reviewed articles have been written. Also, a summary of analytical papers/studies up to the moment have been presented. Most analytical studies use different methods which unfortunately limit the range of reliability of the results. Also, the analytical results can be used to check calculation of range numerical methods. This present

study is further analysis of the effects of thermal radiation on heat and mass heat transfer over an exponentially porous stretching surface (Idowu and Amoo, 2015a).

The resulting governing equations are solved numerically using fourth-order Runge-Kutta method. The aim and scope of this research at this stage is to further analyse the effect of thermal radiation on heat and mass transfer with exponential surface geometry over an exponentially porous surface. The next section is about the materials and methods used in this study

## 2. MATERIALS AND METHODS

Considering a free convective, the boundary layer flow, heat and mass transfer of viscous incompressible fluid over an exponentially stretching surface. The geometry and equations governing the fluid flow of heat and mass transfer is assumed as:

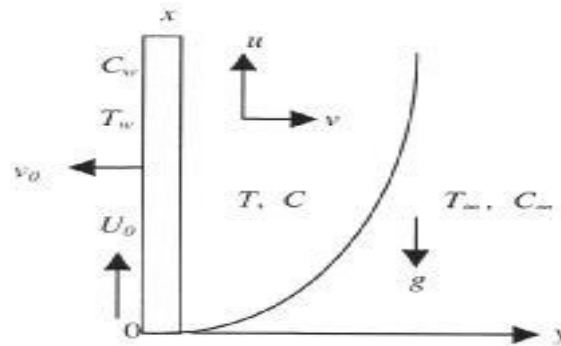


Figure 1: The physical model and coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) \quad (2)$$

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (4)$$

Subject to the following boundary conditions:

$$u = U_0 e^{\frac{x}{L}}, v = -V_0 e^{\frac{x}{L}}, T = T_w = T_\infty + T_0 e^{\frac{x}{2L}}, C = C_w = C_\infty + C_0 e^{\frac{x}{2L}} \quad (5)$$

at  $y = 0$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty$$

Where  $u$ ,  $v$ ,  $C$ , and  $T$  are velocity component in the  $x$  direction, velocity component in the  $y$  direction, concentration of the fluid species, fluid temperature respectively.  $L$  is the reference length,  $U_0$  is the reference velocity,  $V_0$  is the permeability of the porous surface respectively. The physical quantities  $\rho$ ,  $\nu$ ,  $D$ ,  $k$ ,  $C_p$ , are the density, fluid kinematics viscosity, coefficient of mass diffusivity, thermal conductivity of the fluid and specific heat respectively,  $g$  is the gravitational acceleration,  $\beta_T$  and  $\beta_C$  are the thermal and

mass expansion coefficients respectively, then  $q_r$  is the radiative heat flux in the  $y$  direction. By using the Rosseland approximation or constant according to Ibrahim and Suneetha (2015), the radiative heat flux  $q_r$  is given by

$$q_r = -\frac{4\sigma_0}{3\delta} \frac{\partial T^4}{\partial y} \quad (6)$$

Where  $\sigma_0$  and  $\delta$  are the Stefan-Boltzmann approximation and the mean absorption coefficient respectively. Assume the temperature difference within the flow are sufficiently small such that  $T^4$  may be expressed as a linear function of temperature, using Taylor series to expand  $T^4$  about the free stream  $T_\infty$  and neglecting higher order terms, this gives the approximation

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

Introducing the stream function  $u = \frac{\partial \psi}{\partial y}$ ,  $v = -\frac{\partial \psi}{\partial x}$  continuity equation is identically satisfied and equations (2)-(4), gives

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3} + g\beta_r (T - T_\infty) + g\beta_c (C - C_\infty) \quad (8)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \left( \frac{k}{\rho C_p} + \frac{16\sigma_0 T_\infty^3}{3\rho C_p \delta} \right) \frac{\partial^2 T}{\partial y^2} \quad (9)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad (10)$$

The corresponding boundary conditions become:

$$\frac{\partial \psi}{\partial y} = U_0 e^{\frac{x}{2L}}, \frac{\partial \psi}{\partial x} = V_0 e^{\frac{x}{2L}}, T = T_w = T_\infty + T_0 e^{\frac{x}{2L}},$$

$$C = C_w = C_\infty + C_0 e^{\frac{x}{2L}} \text{ at } y = 0 \quad (11)$$

$$\frac{\partial \psi}{\partial y} \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty$$

In order to transform the equations (8), (9) and (10) as well as the boundary conditions (11) into an ordinary differential equations (ODEs), after using the similarity transformations variables (12) with appropriate boundary conditions according to (Sajid and Hayat, 2008),

$$\psi(x, y) = \sqrt{2\nu U_0 L} e^{\frac{x}{2L}} f(\eta), \eta = y \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}}, T = T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta), \quad (12)$$

$$C = C_\infty + C_0 e^{\frac{x}{2L}} \phi(\eta)$$

The following ODEs were obtained.

$$f''' + ff'' - 2f'^2 + Gr\theta + Gc\phi = 0 \quad (13)$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Pr f\theta' - Pr f'\theta = 0 \quad (14)$$

$$\phi'' + Scf\phi' - Scf'\phi = 0 \quad (15)$$

The corresponding boundary conditions take the form:

$$f = f_w, f' = 1, \theta = 1, \phi = 1, \text{ at } \eta = 0, f' = 0, \theta' = 0, \phi = 0 \text{ at } \eta \rightarrow \infty \quad (16)$$

Where  $Pr = \frac{\rho\nu C_p}{k}$  is the Prandtl number,  $R = \frac{4\sigma_0 T_\infty^3}{\delta k}$  is the thermal radiation parameter,  $Sc = \frac{\nu}{D}$  is

the Schmidt number,  $Gr = \frac{g\beta_T(T_w - T_\infty)}{\nu^2}$ , is the thermal Grashof parameter,  $Gc = \frac{g\beta_c(C_w - C_\infty)}{\nu^2}$ ,

is the Solutal Grashof parameter and  $f_w = -V_0 \sqrt{\frac{2L}{\nu U_0}} e^{-\frac{3x}{2L}}$  is the permeability of the plate.

The governing equations of convection, heat and mass transfer in fluids are essentially nonlinear ordinary differential equations. Hence, the system of nonlinear ordinary differential equations together with the boundary conditions are solved numerically using fourth order Runge-Kutta scheme with a shooting technique. The method has been proven to be adequate for boundary layer equations, seen to give accurate results and has been widely used (Stroud, 1996). Therefore, the equations (13), (14) and (15) are non-linear differential equations and its satisfying the boundary conditions (16). The problem is a boundary value problem, applying a shooting technique (guessing the unknown values) to change the conditions to initial value problem. To integrate equations (13) to (15) as an initial value problem we require a value for  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$  but no such values are given in the boundary. The suitable guess values  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$  are chosen and then integration is carried out.

We compare the calculated values for  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$  at  $\eta = 7$  with the given boundary condition  $f''(7) = 0$ ,  $\theta'(7)$  and  $\phi'(7)$  and adjust the estimated values,  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$ , to give a better approximation for the solution. The computations have been performed using a symbolic program and computational computer language MAPLE 18. The step size is taken to be  $\Delta\eta = 0.001$  to satisfy the convergence requirement of  $10^{-5}$  in all cases. The value of  $\eta_\infty$  is noticed to the iteration loop by  $\eta_\infty = \eta_\infty + \Delta\eta$ . The highest value of  $\eta_\infty$  to each parameters are determined when the values of the unknown boundary conditions at  $\eta = 0$  note change to successful loop with error less than  $10^{-5}$ . From the process of numerical computation, the skin-friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to  $\theta'(0)$  and  $\phi'(0)$ , at the plate have been examined for different values of the parameters are presented in a tabular form and discussed. The following parameter values are adopted for computation as default number:  $Gr = Gc = 0.5$ ,  $f_w = 0.0$ ,  $R = 0.01$ ,  $Sc = 0.2$ ,  $Pr = 0.72$ . All graphs are correspond to the value except otherwise indicated on the graph.

Table 1: Effect of  $f_w$ ,  $G_r$ ,  $G_c$ ,  $S_c$ ,  $P_r$ , and  $R$  on  $f''(0)$ ,  $\theta'(0)$  and  $\phi'(0)$  (P-Parameters)

p	Values	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	P	Values	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
$f_w$	0.0	0.12249	1.35877	0.90057	Sc.	0.2	-0.06001	2.20924	0.58364
	0.5	-0.09946	1.72304	1.05342		0.35	-0.21389	2.18116	0.83167
	1.0	-0.40037	2.14593	1.22804		0.62	-0.40037	2.14593	2.22804
	2.0	-1.22861	3.13267	1.64464		1.5	-0.68762	2.09381	2.36611
$G_r$	0.5	-0.84572	3.73355	1.19729	$P_r$	0.72	0.00623	0.78519	1.30062
	3.0	-0.39590	3.76577	1.21612		1.5	-0.20445	1.29415	1.25951
	7.0	0.30193	3.81323	1.24386		3.0	-0.40037	2.14593	1.22804
	10	0.81021	3.84609	1.26297		10	-0.66885	5.55920	1.20162
$G_c$	0.5	-1.19949	3.67811	1.13353	R	0.01	-0.59801	4.10571	1.20673
	1.5	-0.77141	3.73086	1.18773		0.5	-0.48604	2.77260	1.21749
	3.0	-0.20200	3.79270	1.24411		1.5	-0.33183	1.78032	1.23794
	5.0	0.48554	3.85897	1.29954		3.0	-0.18411	1.23269	1.26327

### 3. RESULTS AND DISCUSSION

Table 1 represents the numerical results of variation in Skin friction, Nusselt and Sherwood numbers at the surface with  $f_w$ ,  $G_r$ ,  $G_c$ ,  $S_c$ ,  $P_r$ ,  $R$  which are of physical and engineering interest. From the results it seen that an increase in  $f_w$  decreases the skin friction, Nusselt and Sherwood numbers respectively due to decrease in the boundary layers. Increase in  $G_r$  and  $G_c$  cause increase in the skin friction but decrease the Nusselt and Sherwood numbers. Variations in the values of  $S_c$  decreases the wall layer of skin friction and Sherwood number but increase the Nusselt number. Also, increase in  $P_r$  decreases the skin friction and Nusselt number while it causes increase in the Sherwood number. Moreover,  $R$  increases the boundary layers of the skin friction and Nusselt number but causes decrease in the Sherwood number. Figure 2 shows the effect of thermal Grashof number  $G_r$  on the rate of fluid flow. From the figure, it is seen that increasing in the parameter  $G_r$  leads to an increase in the flow rate, as a result, the velocity boundary layer thickened. Hence, heat has a strong influence on flow rate. The effect of solutant Grashof number on the velocity distribution is illustrated in figure 3. It is noticed from the figure that the velocity increases with an increase in the values of the parameter  $G_c$  due to the fact that  $G_c$  increases free convection current and thereby increases the velocity distribution.

For variation in the values of radiation parameter  $R$ , the dimensionless velocity and temperature profiles are plotted in figures 4 and 5. It is obvious from the distribution that velocity and temperature increases with an increase in the radiation parameter. The effects are as a result of thickness in the momentum and thermal boundary layers.

Figures 6 to 8 exhibit the effect of the suction or injection parameter on the dimensionless velocity, temperature and concentration profiles. It is shown from figure that the suction parameter causes decrease in the velocity indicating the fact that suction stabilizes the boundary layer development, while the injection increases the velocity at the boundary layer indicating that injection supports the flow to penetrate more into the fluid. In figure 7, it is observed that the temperature decreases as the suction parameter increases. This is due to the fact that larger suction leads to faster cooling of the plate, also the temperature increases as the injection parameter increases, this means that heat transfer from the fluid to the surface. Figure 8 represents that the concentration decreases as the suction parameter increases and increases as the injection parameter increases due to respective thinner and thickness in the mass boundary layer.

The influences of the Prandtl number  $P_r$  on the velocity and temperature distribution are presented in figures 9 and 10. It can be noticed from the figures that as the Prandtl number  $P_r$  increases, the dimensionless velocity decreases near the surface and dimensionless temperature also decreases, this is because, as the Prandtl number increases the thickness of the thermal boundary layer decreases and heat is able to diffuse out of the system, hence, the temperature profiles decrease. Figures 11 and 12 depict the behavior of Schmidt number  $S_c$  on the velocity and concentration profiles. Schmidt number can be defined as the ratio of the momentum to the mass diffusivity. It is seen that the velocity and concentration profiles reduce as the Schmidt number increases. Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic velocity and concentration boundary layers. As result of the outcome of this research it can be said that in the era of change in Nigeria, this study has contributed to the fact that sponsoring research in this area and making use of it will help improve the academic researches as well as benefiting the nation.

#### 4. CONCLUSION

This study investigated natural convective boundary layer flow, heat and mass transfer of incompressible viscous fluid over a porous stretching vertical surface in the presence of thermal radiation. The results show that the velocity increases with the increase in the value of radiation  $R$ . The fluid temperature increases with the increase in thermal radiation while fluid temperature decreases with an increase in Prandtl number. The concentration decreases with an increase Schmidt number. It is seen that the velocity and concentration profiles reduce as the Schmidt number increases. Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic velocity and concentration boundary layers. There is need to invest in thermal energy in order to strengthen and tap solar energy to help industries that would develop our economy.

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APPENDIX

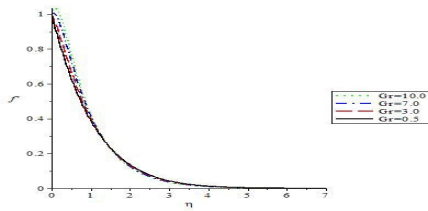


Figure 2 : Velocity Profiles for different values of  $G_r$

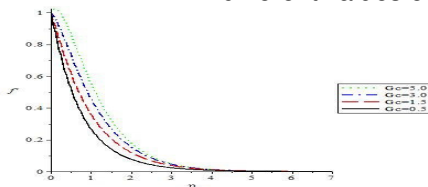


Figure 3 : Velocity Profiles for different values of  $G_c$

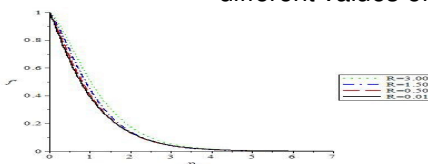


Figure 4 : Velocity Profiles for different values of  $R$

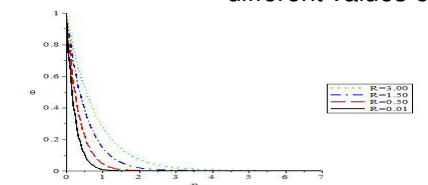


Figure 5 : Temperature Profiles for different values of  $R$

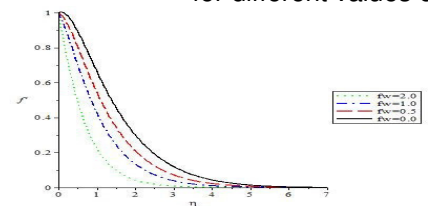


Figure 6 : Velocity Profiles for different values of  $f_w$

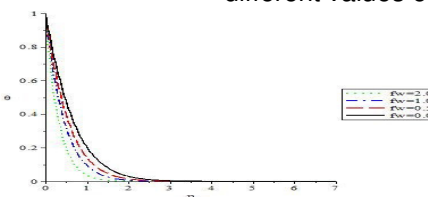


Figure 7 : Temperature Profiles for different values of  $f_w$

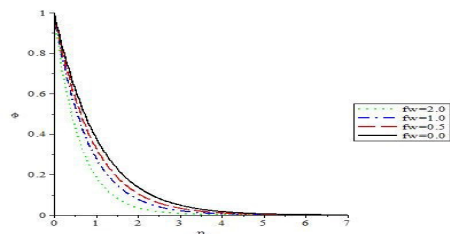


Figure 8 : Concentration Profiles for different values of  $f_w$

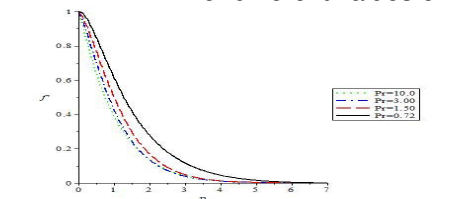


Figure 9 : Velocity Profiles for different values of  $P_r$

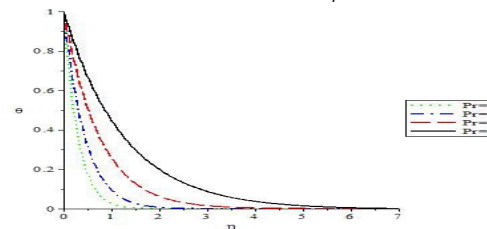


Figure 10 : Temperature Profiles for different values of  $P_r$

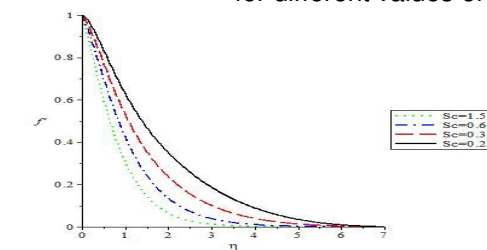


Figure 11 : Velocity Profiles for different values of  $S_c$

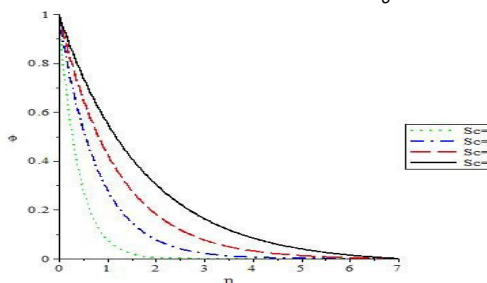


Figure 12 : Concentration Profiles for different values of  $S_c$